M.Sc. PHYSICS - I YEAR

DKP16 : NUCLEAR PHYSICS

SYLLABUS

Unit I : Properties of nucleus and nuclear forces

Quantum properties of nuclear states: Nuclear energy levels nuclear angular momentum, parity, isospin, statistics , nuclear magnetic dipole moment -Nuclear models: Liquid drop model- Bohr-wheeler theory of nuclear fission – Shell model – predictions of shell model – collective nuclear model

Unit II : Nuclear reaction and nuclear decay

Types of nuclear reactions, elastic scattering, inelastic scattering, disintegration, radiative capture, direct reaction – conservation laws – law of conservation of energy, momentum, angular momentum, charge, spin , parity. Nuclear reaction kinematics – Expression for Q-value Nuclear decay: Gamow's theory of alpha decay, Fermi's theory of beta decay – Fermi and Gamow- Teller selection rules – internal conversion – nuclear isomerism

Unit III : Nuclear forces and Properties of nuclear forces

Deuterons – properties of deuteron- ground state of deuteron – excited state – magnetic quadrupole moment of deuteron- neutron-proton scattering at low energies – proton–proton scattering at low energies – meson theory of nuclear forces- reciprocity theorem – Breit-Wigner one level formula

Unit IV : Neutrons

Neutron source - properties of neutron - , charge , spin and statistics, decay, magnetic moment – classification of neutrons – neutron diffusion – neutron current density, neutron leakage rate, thermal neutron diffusion, fast neutron diffusion ,Fermi age equation –nuclear reactors, nuclear chain reaction - four factor formula – general aspects of nuclear reactors – classification of nuclear reactors

Unit V : Nuclear particles

Classification of elementary particles – particle interaction – conservation laws- leptons Hardons-pion - muons – mesons – hyperons - strange particle – CPT theorem -- quark model- Elementary particle symmetries SU(2) and SU(3) symmetry

UNIT I : PROPERTIES OF NUCLEUS AND NUCLEAR FORCES

Quantum properties of nuclear states: Nuclear energy levels nuclear angular momentum, parity, isospin, statistics , nuclear magnetic dipole moment -Nuclear models: Liquid drop model- Bohr-wheeler theory of nuclear fission – Shell model – predictions of shell model – collective nuclear model

Quantum properties of nuclear states

(a) Nuclear Energy Levels : Similar to the discrete energy states of the electrons in an atom, the nucleus have possible energy states given by Heisenberg principle.

$$H_n \psi_n = E_n \psi_n$$

Here Schrodinger wave function ψ_n depends upon spatial coordinates and also spin and isospin quantum numbers. The ground state possesses latent energy, in the form of potential energy which is normalized to zero. Excitation energies of the higher levels are referred to the zero energy of the ground level. Gamma rays of discrete values are emitted by the de-excitation of the nucleus to its ground state. Levels are said to be virtual, if they can dissociate with the emission of a particle and bound if not. The energy level represents excitation energy, nuclear spin J, parity π, lifetime τ, and isospin T.

(b) Nuclear Angular momentum: Atomic nuclei may possess an intrinsic angular momentum as well as a magnetic moment. The nuclear angular momentum can be deduced from the measurement of multiplicity and relative spacing of the spectral lines .

It has been found that the neutron and the proton possess an intrinsic angular momentum referred to as its spin of magnitude ½ ħ. Since nuclei are built up of nucleons (neutrons and protons), each possesses an angular momentum, which consists of both orbital angular momentum due to the motion about the centre of the nucleus and intrinsic spin angular momentum due to the spinning about its axis per nucleon. The total angular momentum state is the resultant of the individual momenta of the constituent nucleons . Corresponding to total angular momentum quantum number J, the absolute magnitude of total angular momentum is $\hbar[J(J+1)]^{1/2}$. The value of J depends on the type of interaction between the nucleons.

LS-Coupling :- In this case the spin –orbit interaction is negligible and there is a collective interaction of orbital and intrinsic momenta ,i,e., J=L ± S,

$$L = \sum_i \ell_i \qquad S = \sum_i S_i$$

For L= 0,1,2,3,...., we have levels ,S,P,D,F ... For each value of L there are (2S+1) possible separated energy levels. The multiplicity (2S+1) is written as a superscript before the letter representing L and the value of J as subscript. Hence for L=1 and S=1/2, we have levels $^2P_{3/2}$ and $^2P_{1/2}$, the spin doublet .

j-j –Coupling :- I n this case the orbital and spin momenta of each individual nucleon are strongly coupled . J is the vector sum of the individual j values .Hence

$$J = \sum_i j_i \text{ , where } j_i = \ell_i + s_i$$

This type of coupling is called strong spin –orbit coupling. If s-nucleon (ℓ=0, j=1/2) couples with p-nucleon (ℓ=1,j=3/2,1/2) .

The total angular momentum of a nucleus is usually called nuclear Spin. Experimentally it is found that all nuclei have relatively low spin of the ground state by integral multiples of ħ .The term "spin of the nucleus," without any specification ,always refers to the ground state . It has been found that all even –even nuclei have a spin J=0 in the ground state . Odd-odd nuclei all have integral nuclear spin, other than zero . All odd-even nuclei have half integral nuclear spins.

Total angular momentum vector J can be oriented in space with respect to a given axis in (2J+1) directions. The component along the axis in any of the states has the magnitude mħ where m is the magnetic quantum number ,having the values from J to –J, as (J-1),(J-2) ……….,-(J-1). Thus the largest value of m is ℓ.

c) Parity : Nuclear parity is the product of the parities of the nuclear constituents. For even - even nuclei the ground states have positive parity . The parity of the odd nuclei is given by $(-1)^{\ell}$ where ℓ is the orbital angular momentum of the unpaired nucleon. Odd-odd nuclei have a parity $(-1)^{\ell_n + \ell_p}$. parity is even if this sum is positive and odd if the sum is negative.

d) Isospin : For a given nucleus the value of T_3 or T_z is just the minus on half of the neutron excess. $T_3 = -1/2\ (N-Z) = -1/2\ (A-2Z)$

In a set of isobars of given A, a member X will have an isospin $T_{z\ max}$ largest among the set. For this T, $T_{Z\ max} = 1/2\ (Z_X - N_X)$, having (2T+1) states. These states are corresponding to different T_z and hence to different charges ($Z=A/2 + T_z$) . The isobars ^{14}C,^{14}N,^{14}O have

T_z = -1, 0, +1 respectively.

The mirror nuclei ^3H and ^3He have T=1/2 in their ground state.

e) Statistics

The statistics is mainly categorizes as Bose Einstein and Fermi Dirac statistics.

The nuclei with even mass number and photons obey Bose Einstein and the nuclei with odd mass number obey Fermi Dirac statistics. ^1H, ^7Li, ^{19}F, ^{23}Na, ^{31}P, ^{35}Cl obey Fermi Dirac statistics. ^2H, ^4He, ^{12}C, ^{14}N, ^{16}O, obey Bose Einstein statistics .

Nuclear magnetic dipole moment

The protons inside the nucleus are in orbital motion and therefore produce electric currents which produce nuclear magnetic field. Each nucleon possesses an intrinsic magnetic moment which is parallel to its spin and is probably caused by the spinning of the nucleon. A spinning positive charge produces a magnetic field whose N-pole direction is parallel to the direction of spin. The magnetic moment is defined as positive in this case.

If a particle having a charge q and mass m circulates about a force centre with a frequency ν, the equivalent current $i=q\nu$. From Kepler's law of areas, the area swept dA in time dt by the particle is related with its angular momentum ℓ as

$$dA/dt = \tfrac{1}{2}\ \ell/m = constant$$

On integration over one period T, we get

$$A = \tfrac{1}{2}T\ \ell/m$$

The magnetic moment of the ring current around the area of magnitude A is called the orbital magnetic dipole moment and is given by

$$\mu_l = iA = q\nu T\,\ell/2m = \frac{q}{2m}$$

μ $_l$ and l are proportional to each other . This relation is valid in quantum mechanics.

Since electron, proton and neutron posses a spin in addition to angular momentum experimentally it is found that the spin is a source of magnetic moment. Therefore using q=e and dimensionless correction factor g_s we can write

The spin magnetic dipole moment as

$$\mu_s = g_s (e/2m) s.$$

The factor g_s is different for the electron, proton and neutron. Similarly, we introduce a factor g_ℓ for the orbital angular momentum and have

$$\mu_\ell = g_\ell (\mathrm{e}/2\mathrm{m})\,\ell.$$

The total magnetic dipole moment μ is given as:

$$\mu = \mu_s + \mu_\ell = (e/2\mathrm{m})[g_s s + g_\ell \ell]$$

For the nucleus of mass number A, magnetic dipole moment, called nuclear magnetic moment.

$$\mu = \frac{e}{2\mathrm{m}}\left[\sum_{k=1}^{A} g_s s_k + \sum_{k=1}^{Z} g_\ell \ell_k\right]$$

Since total momentum of the nucleus

$$J = \sum_{k=1}^{Z} \ell_k + \sum_{k=1}^{A} s_k$$

$$\mu = g_J (e/2\mathrm{m}) J = g_J (e\hbar/2\mathrm{m}) J/\hbar,$$

where g_I is the gyromagnetic ratio (g-factor) of the nucleus. It is the dimensionless ratio of the magnetic moment μ in terms of $e\hbar/2m$ to the angular momentum J in terms of $\hbar$. If ℓ is measured in terms of $\hbar$, it is called angular momentum quantum number. Using quantum mechanics we have [$\ell(\ell+1)$]½ and [s(s+1)] ½ instead of ℓ and s respectively, we thus get

$$g_J = \frac{1}{2}(g_\ell + g_s) + \frac{1}{2}(g_\ell - g_s)\frac{\ell(\ell+1) - s(s+1)}{J(J+1)}$$

The magnetic dipole moment is measured in terms of nuclear magneton, defined as

$$\mu_N = e\hbar/2\mathrm{m}_p = 5.05079 x 10^{-27} A.m^2$$

$$= 3.15245 x 10^{-8} eV - m^2/weber$$

g_ℓ = 1 for proton	gs = 5.85 for proton
=0 for neutron	= -3.826 for neutron

Liquid Drop Model

The constant density of the nuclear matter and the constant binding energy per nucleon are very similar to those found in a liquid drop. The very strong short range interaction between the nucleon permits us to consider their collective behavior in determining the properties of the nucleus.

There are reasons to believe that each individual molecule within a liquid drop exerts an attractive force upon a group of molecules in its immediate neighborhood. The force of interaction does not extend to all the molecules within the drop. This is known as the saturation of the force. In order to calculate the potential of the interaction, it is necessary to know the number of interacting pairs of molecules within the drop.

The binding energy BE of a nucleus is proportional linearly to the number of nucleus within it, so that the binding fraction f_B is linearly constant for most nuclei. This fact shows a close resemblance of the nucleus with a liquid drop. Thus we come to the conclusion that the inter nucleon force within the nucleus attains a saturation value, so that each nucleon can interact only with a limited number of nucleon in its close vicinity. Apart from this, there are certain other points of resemblance between the nucleus of an atom and a liquid drop:

1. The nuclear force is similar to the force of surface tension on the surface of the liquid drop.
2. As in the case of a liquid drop, the density of the nuclear matter is independent of its volume. The nuclear radius R_0 $A^{1/3}$ where A is the mass number. Hence the nuclear volume VA. Since the nuclear mass M~A, the density of the nuclear matter ρ_m = M/V is independent of A.

$$\text{Density} = \frac{\text{Mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{M}{\frac{4}{3}\pi (R_0 A^{\frac{1}{3}})^3} = \frac{3}{4\pi R_0{}^3}$$

3. The different types of particles, e.g., neutrons, protons, deuterons, α-particles etc. are emitted during nuclear reactions. These processes are analogous to the emission of the molecules from the liquid drop during evaporation.
4. The internal energy of the nucleus is analogous to the heat energy within the liquid drop.

5. The formation of the short lived compound nucleus by the absorption of a nuclear particle in a nucleus during a nuclear reaction is analogous to the process of condensation from the vapour to the liquid phase in the case of the liquid drop.

The liquid drop model is not very successful in describing the low lying excited states of the nucleus. Because of the collective motions of the large number of nucleons involved, the model gives rise to closely spaced energy levels. Actually however, these are found to be quite widely spaced at low excitation energies

Deformation of liquid drop

The fission process can be explained with the help of liquid drop model. The incident neutron combines with the nucleus to form highly energetic compound nucleus. Its extra energy is partly the kinetic energy of the neutron but largely the added binding energy of the incident neutron. This energy appears to initiate a series of rapid oscillations in the drop, which tend to distort the spherical shape so that the drop may become ellipsoidal in shape. The surface tension forces tend to make the drop return to its original spherical shape, while the excitation energy tends to distort the shape still further. If the excitation energy is sufficiently large, the drop may attain the shape of a dumb-bell. If the oscillations become so violent that the critical state, stage fourth of Figure, is reached then the final fission into stage fifth is inevitable. Thus there is a *threshold energy* or a *critical energy* required to produce stage fourth after which the nucleus cannot return to stage first. When the distortion produced is not pronounced enough to get the nucleus beyond the critical point, the ellipsoid will return to the spherical shape with the excitation energy being liberated in the form of ϒ-rays and we have a radiative capture rather than fission.

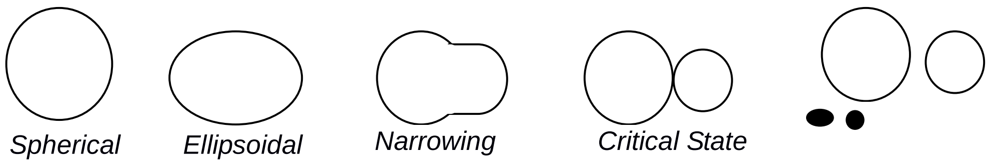


Figure : Schematic representation of nuclear fission

Bohr and Wheeler's Theory of Nuclear fission

The potential energy of the drop in the different stages can be calculated as a function of the degree of deformation of the drop. The potential energy is plotted against r, the separation of the centers of two fission fragments. The curve is supposed to be divided into three regions.

In region I, the fragments are completely separated and their potential energy E is simply the electrostatic Coulomb energy resulting from the mutual repulsion of the two positively charged nuclear fragments. If distance $r=2R$, when the drops just touch each other, energy E at the point is less than the corresponding Coulomb potential by an amount CD. This amount is equal to the potential of the surface forces which are just beginning to come into play at this point. As we pass through region *II*, we reach the critical distance r_c, where the potential energy curve has a maximum value E_b. This corresponds to barrier height and explains why fission does not take place spontaneously in all cases where $E_f > 0$. An additional amount of energy $E_a = E_b - E_f$ the activation energy is required by the nuclear system before the potential barrier can be surmounted and fission can take place. In the III region, the fragments have coalesced and the short range nuclear forces have become predominant.

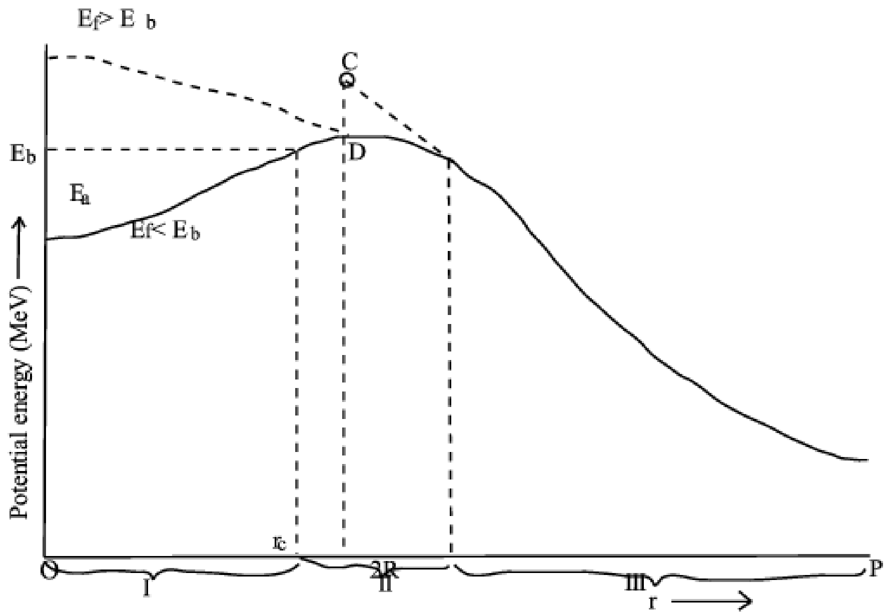


Figure : Potential energy curve of Nuclear fission

The first theoretical treatment of this process was carried out by Bohr and Wheeler. They applied a simple form of analysis(Legendre polynomial expansion) to express the radius *r* making angel θ with the axis of maximum deformation

$$r = R\left[1+\sum_{l=0}^{\infty}\alpha_l P_l(\cos\theta)\right] = R[1+\alpha_2 P_2(\cos\theta)+\alpha_3 P_3(\cos\theta)+...] \quad \text{---------(1)}$$

where R is the radius of the spherical nucleus and α_2, α_3 are the deformation parameters.

Here $\alpha_0 = \alpha_1 = 0$, as the centre of mass of the drop is assumed to remain unchanged.

The surface energy of a spherical drop $Es=4\pi R^2T=4\pi\ [R_0A^{1/3}]^2T$, where A is the mass number and T is the surface tension. Hence surface energy of the deformed drop in terms of deformation parameters is given by

$$Es = 4\pi R_0^2 A^{2/3}T\left[1+\alpha_2\ (\frac{3}{2}.\cos^2\theta-\frac{1}{2}+..........\right].$$

$$Es = 4\pi R_0^2 A^{2/3}T\left[1+\frac{2}{5}\alpha_2^2+\frac{5}{7}\alpha_3^2+...\right].$$

The change in surface energy of the drop due to deformation is

$$\Delta E_s = E_s\left[\frac{2}{5}\alpha_2^2+\frac{5}{7}\alpha_3^2+...\right]. \quad ....(2)$$

The Coulomb energy of a spherical drop $E_c = \frac{3}{5}Z^2e^2/4\pi\varepsilon_0 R$, hence that of the deformed drop

$$E_c = \frac{3}{5}\frac{Z^2e^2}{4\pi\varepsilon_0 R_0 A^{1/3}}\left[1+\alpha_2\left(\frac{3}{2}\cos^2\theta-\frac{1}{2}\right)+...\right]^{-1}$$

$$\Delta E_c = E_c\left[-\frac{1}{5}\alpha_2^2-\frac{10}{49}\alpha_3^2+...\right] = -E_c\left[\frac{1}{5}\alpha_2^2+\frac{10}{49}\alpha_3^2+...\right] \quad ....(3)$$

Thus the total energy variation, keeping only the α_2^2 term is

$$\Delta E=\Delta E_s+\Delta E_c=\frac{1}{5}\alpha_2^2[2E_s-E_c]$$

If it is positive, i.e., $2E_s > E_c$ the drop is stable to small distortions. Fissions may occur spontaneously if ΔE is negative or $E_s < \frac{1}{2}E_c$.

$$4\pi R_0^2 A^{2/3} T < 3Z^2 e^2 / 40\pi\varepsilon_0 A^{1/3} R_0 \text{ or } Z^2 / A > 45$$

The ratio $E_c / 2E_s$ is known as critical parameter, represented by χ . Thus when $\chi < 1$, the nucleus is stable against spontaneous fission. It is possible to estimate the degree of distortion of a nucleus in the critical state by equating the critical or threshold energy E_{th} to the total energy variation ΔE From semi-empirical data $4\pi R_0^2 T = 13 MeV$,hence for

^{238}U; Es =520MeV and $E_c = 830 MeV$ thus $\alpha_2^2 = 1/7$.

Threshold energy or critical energy is the energy that has to be imparted to the nucleus in order to reach this critical shape when the deformed drop is about to split into two equal drops. The threshold energy is given as

$$E_{th} = 4\pi R^2 T f(\chi) = 4\pi R_0^2 A^{2/3} T f(\chi) \qquad \ldots(4)$$

This energy can be calculated by neglecting the second order change in energy due to the neck joining the two fragments.

$$E_{th} = 2(4\pi R_0^2) T\left(\frac{1}{2}A\right)^{2/3} - 4\pi R_0^2 A^{2/3} T + 2\text{x}\frac{3}{5}\text{x}\left(\frac{1}{2}Ze\right)^2 / \ 4\pi\varepsilon_0 R_0 \left(\frac{1}{2}A\right)^{1/3}$$

$$+\left(\frac{1}{2}Ze\right)^2 / 8\pi\varepsilon_0 R_0 \left(\frac{1}{2}A\right)^{1/3} - \frac{3}{5}(Ze)^2 4\pi\varepsilon_0 R_0 A^{1/3}.$$

$$E_{th} / 4\pi R_0^2 T A^{2/3} = f(\chi) = 0.260 - 0.215\chi.$$

For an uncharged droplet *=0* and *f(0)=0.260*, hence there are no electrostatic forces and the critical energy is just the work done against surface tension in separating into two drops. For $\chi = 1$, a small deformation from the spherical shape causes the drop to reach the critical shape and to separate.

If the critical energy is compared with the excitation energy, it becomes possible to predict fission probability. The excitation energy E_e, contributed to the resultant compound nucleus by the capture of a neutron, is equal to the binding energy of neutron in the compound nucleus and can be calculated by the relation.

$$Ee = B(A+1, Z) - B(A,Z) = {}^{A}_{Z}M + M_n - {}^{A+1}_{Z}M.$$

The values of the excitation energy calculated in this way for a number of heavy nuclei are listed in the table and compared with the corresponding values of the critical energy. In reviewing the results, it is seen that for ^{238}U a critical deformation energy of 6.5 *MeV* is necessary for fission, but it acquires only 5.9 *MeV* when it takes up a neutron of zero K.E. Thus no fission is possible with thermal neutrons with 0.03 *eV* energy. If the neutrons have a K.E. of 0.6 *MeV* fission becomes possible. Experiments indicate that neutrons of about 1*MeV* energy are required. The fission cross section increases rapidly with neutron energy. The situation is quite different with ^{235}U. Here the excitation energy or the energy available by the capture of a slow neutron is greater than the threshold energy. It is evident that in this case thermal neutrons should be capable of causing fission of ^{235}U nucleus.

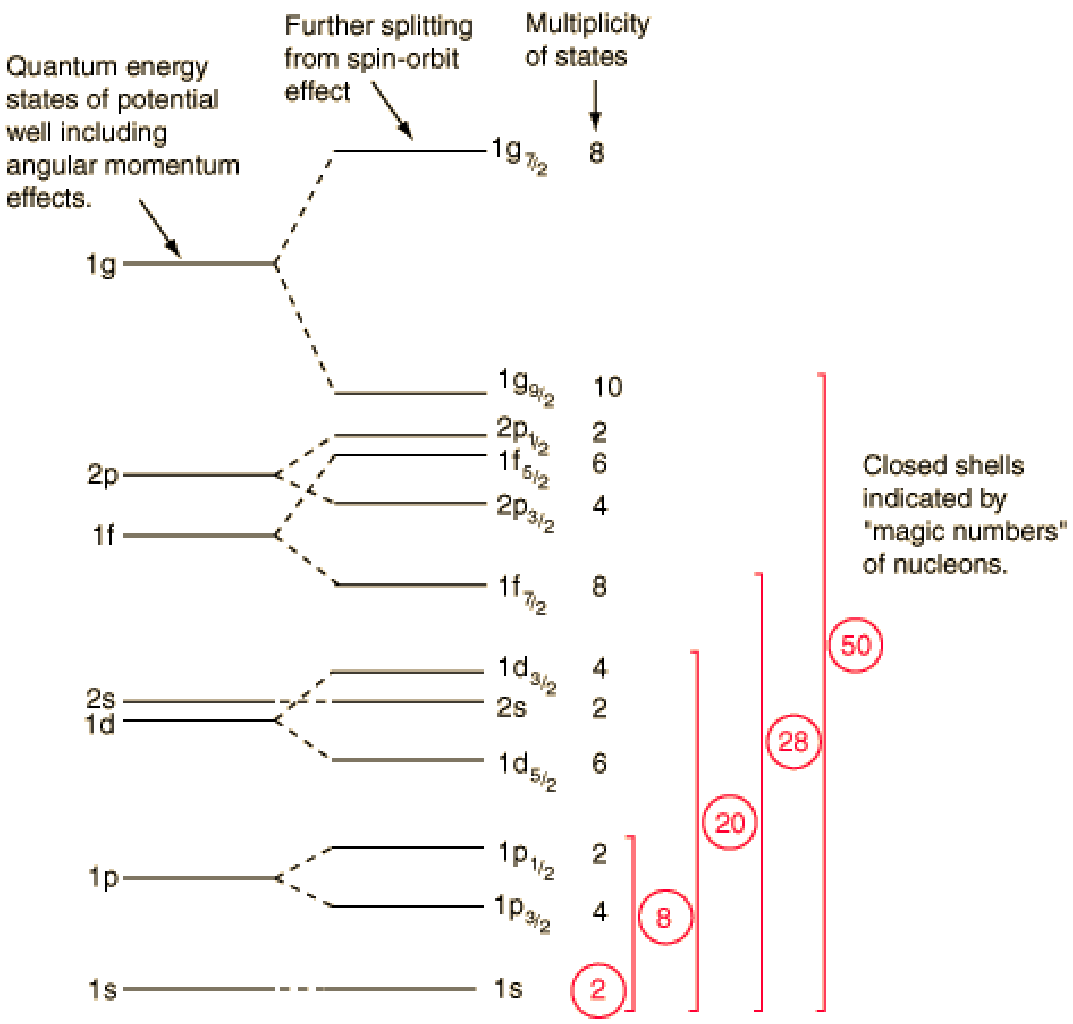
Table : Excitation Energy and Critical energy for some Nuclides

Compound Nucleus	E_e*(MeV)*	E_{th}*(MeV)*	E_e-E_{th} *(MeV)*
^{232}Pa	5.4	5.0	0.4
^{233}Th	5.1	6.5	-1.4
^{235}U	6.6	5.5	1.1
^{238}Np	6.0	4.2	1.8
^{238}U	5.9	6.5	-0.6
^{240}Pu	6.4	4.0	2.4

Shell Model of Nucleus

It is believed that protons and neutrons in a nucleus to be in a continuous process of collision with each other. With the enormous strong force acting between them and with so many nucleons to collide with, how can nucleons possibly complete whole orbits without interacting. According to Pauli's exclusion principle, no two electrons cannot occupy the same quantum state. The evidence for a kind of shell structure and a limited number of allowed energy states suggests that a nucleon moves in some kind of effective potential well created by the forces of all the other nucleons. This leads to energy quantization in a manner similar to the square well Potential . The labels on the levels are somewhat different from the corresponding symbols for atomic energy levels. The energy levels increase with orbital angular momentum quantum number *l*, and the s,p,d,f... symbols are used for *l* =0,1,2,3... just like the atomic case. But there is really no physical analog to the principal quantum number n, so the numbers associated with the level just start at n=1 for the lowest level associated with a given orbital quantum number. In addition to the dependence on the details of the potential well and the orbital quantum number, there is a sizable spin-orbit interaction which splits the levels by an amount which increases with orbital quantum number. This leads to the overlapping levels as shown in the illustration.

The subscript indicates the value of the total angular momentum j, and the multiplicity of the state is 2j + 1. The contribution of a proton to the energy is somewhat different from that of a neutron because of the coulomb repulsion, but it makes little difference in the appearance of the set of energy levels. It is found that nuclei with even numbers of protons and neutrons are more stable than those with odd numbers. In particular, there are "magic numbers" of neutrons and protons which seem to be particularly favored in terms of nuclear stability, they are : 2,8,20,28,50,82,126. Nuclei which have both neutron number and proton number equal to one of the magic numbers can be called "doubly magic", and are found to be particularly stable.



Predictions of the Shell Model

1. ***Stability of the closed shell nuclei*:** This scheme clearly reproduces all the magic numbers, 2,8,20, 28,50,82,126

2. ***Spins and Parities of Nuclear Ground States*:** The shell model has been very successful in predicting the ground state spin of a large number of nuclei. According to this model the neutron and proton levels fill independently. There are following rules for the angular momenta and parities of ground states.

   (i) Even-even nuclei have total ground state angular momentum $J=0^+$. There is no known exception to this rule.

(ii) With an odd number of nucleons, i.e. a nucleus with odd Z or odd N, the nucleons pair off as far as possible so that the resulting orbital angular momentum and spin direction are just that of the single odd particle.

(iii) An odd-odd nucleus will have a total angular momentum which is the vector sum of the odd neutron and odd proton j-values. The parity will be the product of the proton and neutron parities, i.e. parity = $(-1)^{ln+lp}$.

We expect from first rule that the angular momentum is zero not only for ${}^{4}_{2}He$ and ${}^{16}_{8}O$ but also for ${}^{88}_{38}Sr, {}^{192}_{76}Os, {}^{238}_{92}U$ and all the other even-even nuclei. Some actual examples of odd even nuclei are now presented. Consider the nucleus ${}^{13}_{6}C$. The six protons and six of the seven neutrons are paired up in the configuration $1s^{(2)}_{1/2}\ 1p^{(4)}_{3/2}$. The odd neutron is in the $1s_{1/2}$, designation. The ground state angular momentum indicated as the subscript in $p_{1/2}$ i.e. ½, a value which is observed experimentally. For nucleus ${}^{13}_{7}N$, the unpaired particle is a proton with spin ½. As a second example consider ${}^{17}_{8}O$ and ${}^{17}_{9}F$. The shells are filled according to $1s^{(2)}_{1/2}1p^{(4)}_{3/2}1p^{(2)}_{1/2}1d^{(1)}_{5/2}$

If it is ${}^{17}O$, the last unpaired nucleon is a neutron and has a spin 5/2: if it is ${}^{17}F$, the last particle is a proton with spin 5/2. Thus the model predicts 5/2 which is also the observed value for the ground state spin for each of these nuclei.

3. Magnetic Moments of Nuclei: In an odd nucleus, the total angular momentum J of the nucleus is equal to the angular momentum j of the last unpaired nucleon. Thus we see that magnetic moment of the nucleus is produced by the odd nucleon only. The orbital angular momentum with numerical value $\sqrt{l(l+1)}$ and the spin s with numerical value $\sqrt{s(s+1)}$ couple to a total angular momentum J with numerical value $\sqrt{j(j+1)}$, in units of ħ . The magnetic moment associated with spin angular momentum s is given by $\mu_s = g_s s$.

Similarly the magnetic moment associated with orbital angular momentum l is given by $\mu_l = g_l l$.

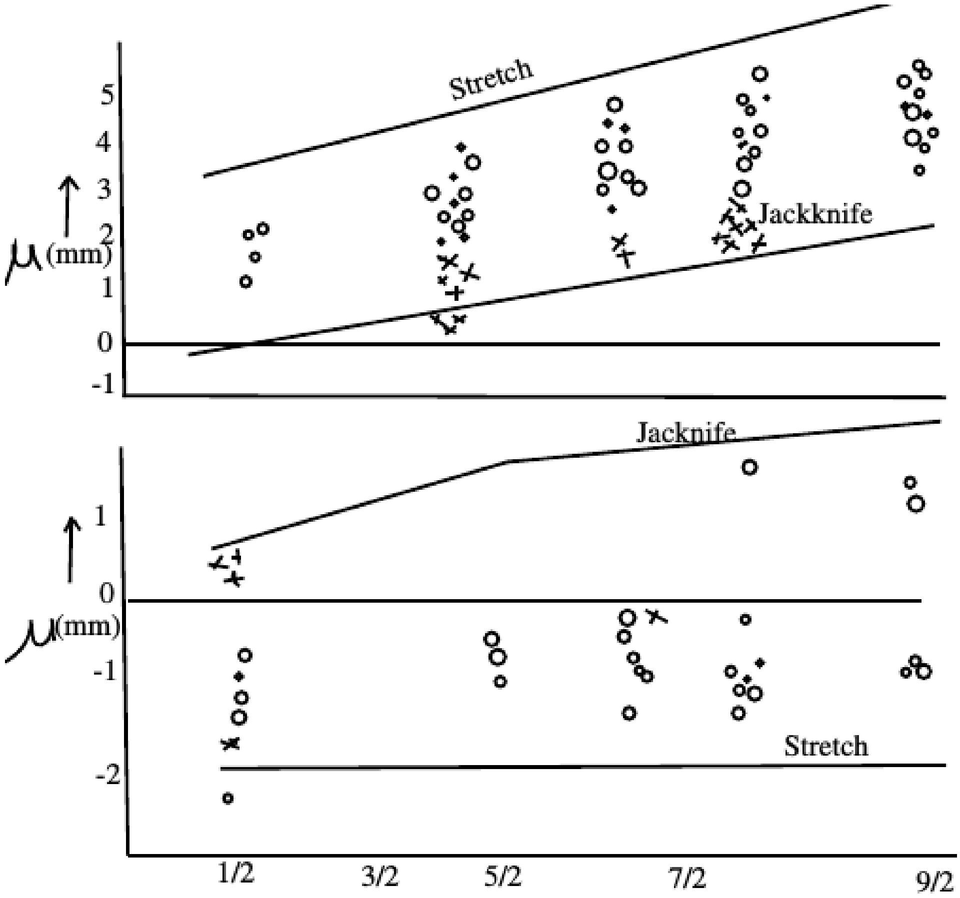


Figure: Magnetic dipole moments against angular momentum- Schmidt lines

(above) for nuclei of odd Z-even N and (below) for nuclei of even Z-odd N

Hence μ = sum of the components of the vectors $g_l l$ and $g_s s$ along the *j*. By applying the cosine rule to the triangle formed by the *l*, *s* and *j*, the above relation can be written as

$$\mu = g_l \sqrt{[l(l+1)]}\frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{[l(l+1)j(j+1)]}} + g_s \sqrt{[s(s+1)]}\frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{[s(s+1)j(j+1)]}}$$

$$= \frac{j(j+1)+l(l+1)-s(s+1)}{2\sqrt{[(j(j+1)]}} g_l + \frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{[j(j+1)]}} g_s$$

Since for a single particle, the spin *s* = ½ and there are two possible cases.

l parallel to s (Stretch case) ; J = *l* + *s* = *l* + ½ .

l antiparallel to s (Jacknife case) ; J = *l* – *s* = *l* – ½ .

Hence

$$\mu = \left(J - \frac{1}{2}\right)g_l + \frac{1}{2}g_s \qquad \text{for stretch case} \quad \text{-----(2)}$$

$$\mu = \frac{J}{J+1}\left[\left(J + \frac{3}{2}\right)g_l - \frac{1}{2}g_s\right] \qquad \text{for Jackknife case----(3)}$$

These relations define two curves, for μ versus J, with the values J = l ± ½, for each class of odd even nucleus. The values of μ are known as the Schmidt value and the curves are known as Schmidt lines. When we substitute the above equations (2) and (3) the g factors which correspond to single nucleons are

g_l = 1 and g_s = 5.58 for protons and g_l = 0 and g_s = -3.82 for neutrons.

Collective Nuclear Model

The shell model is based on the assumption of the existence of a spherically symmetric potential in the nucleus, plus a spin-orbit coupling term. The different types of coupling of the angular momenta assumed for the loose nucleons outside the core given rise to the different forms of the shell model.

The shell model, with some refinements, has been successfully applied to explain many features of the nucleus in the ground state and in some of the excited states. However, it fails in explaining the observed large electric quadrupole moments (Q) of the nuclei in many cases and the quadrupole transition. In such cases where Q is n times the single particle value, we must assume that 2n particles are involved in producing the observed Q since the neutrons cannot directly contribute to Q. It is the collective motion of fairly large number of nucleons which determines the large values of Q for nuclei far from closed shells.

To explain these failures of the shell model by introducing the idea of deformation in the shape of the nuclear core due to the motion of the loose odd nucleon outside the core in odd A nuclei. Such deformation would cause the quadruple moment to be higher than the single particle value. Transition rate is also increased. Further elaborated the model, combining the single particle and collective motions into a unified model which gave a more complete description of the deformed nuclei.

In nearly spherical nuclei, the coupling between the collective motion of the nucleons in the core and the motion of the loose nucleons outside the core is weak. On the other hand, for strong coupling, the surface is distorted and the potential felt by the loose particles is not spherically symmetric. These particles, moving in a non-spherically symmetric shell model potential, maintains the deformed nuclear shape. The situation is similar to that in a linear molecule, we can then write the total energy as the sum of the rotational, vibrational and nucleonic energies of the nucleus, as in the case of the molecule. In the present case, the nucleonic energy replaces the electronic energy of the molecules.

$E_{tot} = E_{rot} + E_{vib} + E_{nuc}$

The collective motion of the nuclear core gives rise to the rotational and vibrational term, while nucleonic energy term is due to the motion of the loose nucleons.

Mathematically this means that E_{tot} is composed of three additive parts containing rotational energy state, vibrational energy state and nucleonic energy state. The total energy function is the product of three functions each containing the respective energy functions . The vibrational energy states of nuclei are found by flexing of nuclear surface and complex nature. The nuclear rational motion is also somewhat complex in that it is not a rigid body rotation . But a rotation of shape of the deformed surface enclosing free particles. The collective motion now becomes a vibration about the equilibrium shape, and a rotation of the nuclear orientation which maintains the deformed shape. This explain the collective model of the nucleus

UNIT II : NUCLEAR REACTION AND NUCLEAR DECAY

Types of nuclear reactions, elastic scattering, inelastic scattering, disintegration, radiative capture, direct reaction – conservation laws – law of conservation of energy, momentum, angular momentum, charge, spin , parity. Nuclear reaction kinematics – Expression for Q-value Nuclear decay: Gamow's theory of alpha decay, Fermi's theory of beta decay – Fermi and Gamow- Teller selection rules – internal conversion – nuclear isomerism

Types of Nuclear Reactions

Depending upon the product nuclei, the nuclear reactions can be categoried. The artificial transmutation of a nucleus produced in the pioneering experiment of Rutherford is a type of nuclear reaction. Various types of nuclear reactions have since been produced. These can be conveniently classified as below.

1. Elastic Scattering

The incident particle strikes the target nucleus and leaves without energy loss but in general with altered direction of motion.

Scattering of α particle in gold is a good example of this process.

$$^{4}_{2}He+^{197}_{79}Au\rightarrow^{4}_{2}He+^{197}_{79}Au$$

2. Inelastic scattering

The scattered particle may loss KE. This being corresponding increase in the internal energy of the nucleus which is excited to a high quantum state. This inelastic scattering can be represented by the well known example.

$$^{7}_{3}Li+^{1}_{1}H\rightarrow^{7}_{3}Li+^{1}_{1}H$$

The star * indicates that after scattering nucleus is left in an excited state.

3. Radiative capture

The particle may combine with a nucleus to produce a new nucleus or a compound nucleus which is in an excited state. The excess energy is emitted in the form of γ ray photon. This type of process is known as radiative capture.

$$^{26}_{12}Mg+^{1}_{1}H\rightarrow^{27}_{13}Al+\gamma$$

4. Disintegration process

The first nuclear transmutation observed by Rutherford is an example of this process $^{14}N\ (\alpha, p)\ ^{17}O$. On striking the target nucleus the incident particle is absorbed and a different particle is ejected. The product nucleus differs from target nucleus

$$^{14}_{7}N+^{4}_{2}He\rightarrow^{17}_{8}O+^{1}_{1}H$$

6. Photo – disintegration

In this case the target nucleus is bombarded with very high energy γ-rays, so that it is raised to an excited state by the absorption of the latter. If the energy is high enough, one or more particles may be liberated.

The reaction can be written as X (γ, y) Y.

$$ {}_1^1H + \gamma \rightarrow {}_1^1H + {}_0^1n $$

7. **Elementary particle reactions**

These involve either the production of elementary particles other than nucleons or nuclei as a result of the reaction or their use as projectiles or both of these. Examples are

$$p + p \rightarrow p + n + \pi^+;$$

$$\pi^+ + p \rightarrow \pi^0 + n;$$

$$p + \pi \rightarrow K^0 + \Lambda^0 \ .$$

This reactions are usually produced at extremely high energies which may be several hundred MeV or more.

Conservation Laws in Nuclear Reactions

In any nuclear reaction, certain quantities must be conserved. We shall merely list the various conservation laws that appear to be valid in ordinary nuclear interactions.

(i) Conservation of mass number

The total number of neutrons and protons in the nuclei taking part in a nuclear reaction remains unchanged after the reaction. Thus in the reaction X(x,y)Y represents the sum of mass numbers of X and x must be equal to the sum of the mass numbers of Y and y:

$$A_X + A_x \rightarrow A_Y + A_y \qquad \dots (1)$$

Where A_X, A_x, A_Y, A_y are the mass number of X, x, Y, y respectively.

In the general case of reactions involving elementary particles the law can be expressed by requiring the total number of heavy particles remains unchanged in a reaction.

(ii) Conservation of atomic number

The total number of protons of the nuclei taking part in a nuclear reaction remains unchanged after the reaction. This means that the sum of atomic numbers of X and x(Z and z) is equal to the sum of atomic numbers of Y and y (Z′ and z′)

$$Z + z = Z' + z' \qquad \dots (2)$$

(iii) Conservation of energy

In order to apply the law of conservation of energy in the case of a nuclear reaction, it is necessary to take into account the mass-energy equivalence predicted by the special theory of relativity. Conservation of energy requires that the total energy, including the rest-mass energies of all the nuclei taking part in a reaction and their kinetic energies, must be equal to the sum of the rest-mass energies and the kinetic energies of the products.

(iv) Conservation of linear momentum

If p_X, p_x, p_Y and p_y represent the momentum vectors of the different nuclei taking part in a reaction, then the law of conservation of linear momentum gives

$$p_X + p_x = p_Y + p_y \qquad \dots (4)$$

Eq.(4) holds in an arbitrary frame of reference. In the laboratory frame of reference in which the target nucleus is at rest $p_X = 0$ and the above equation becomes

$$p_x = p_Y + p_y \qquad \dots (5)$$

In the frame of reference in which the centre of mass of the two particles before collision is at rest, we have to write $p_X + p_x = 0$, which gives $p_Y + p_y = 0$ i.e., the centre of mass of the product parallel is also at rest in this system.

(v) Conservation of angular momentum

In a nuclear reaction of the type X + x→ Y + y, the total angular momentum of the nuclei taking part in the reaction remains the same before and after the reaction.

(vi) Conservation of parity

The strong interaction in which parity is conserved, the parity before the reaction must be equal to the parity after the reaction. Parity conservation results in certain selection rules, which limit the possible nuclear reactions that may occur starting from a given initial state .

(vii) Conservation of isotopic spin

Denoting isotopic spin vectors for the initial and final states by T_i and T_f, we have from the law of conservation of isotopic spin applicable in the case of strong interaction.

$$T_i = T_f$$

Since for the reaction X + x → Y + y, $T_i = T_X + T_x$ and $T_f = T_Y + T_y$

We have $T_X + T_x = T_Y + T_y$

Isotopic spin is a characteristic of the nuclear level. Hence the above conservation law can be used to identify the levels of the nuclei produced in the reaction. In particular if $T_x = T_y = 0$, we must have $T_X = T_Y$.

Nuclear Reaction Kinematics

The conservation of energy and momentum imposes certain restriction on the reactions. These restrictions are called kinematic restrictions and this mathematical method is known as kinematics.

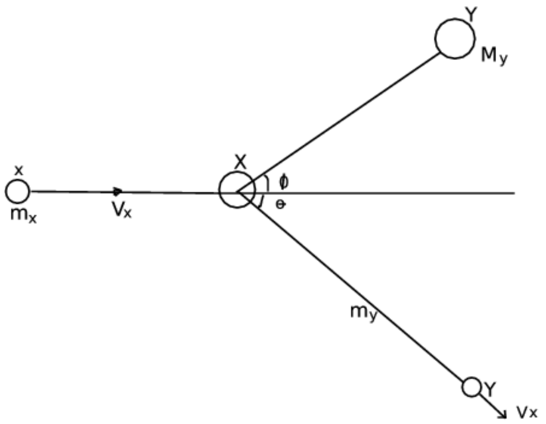


Figure :Schematic diagram of nuclear reaction

Consider a nuclear reaction

$X + x \rightarrow Y + y,$

When X, x, Y and y are the target nucleus, bombarding particle, product nucleus and product particle, respectively. It will be assumed that target nucleus is at rest so it has no kinetic energy. Since total energy is conserved in a nuclear reaction, therefore, we get

$$M_X c^2 + (E_x + m_x c^2) = (E_Y + M_Y c^2) + (E_y + m_y c^2) \qquad ......(2)$$

Where m_x, M_X, m_Y, M_Y all represent respective masses of incident particle, target nucleus, product particle and product nucleus. We now introduce a quantity Q which represents the difference between the kinetic energy of the products of reaction and that of the incident particle.

$$Q = E_Y + E_y - E_x \qquad \text{------------------------------------}(3)$$

From Equations (2) and (3), we have

$$(M_X + m_x – M_Y – m_y)\, c^2 = Q$$

The quantity Q is called the energy balance of the reaction or more commonly Q value of the reaction. If Q *is + ve* the reactions is said to be exoergic. This occurs if sum of the masses of incident particle and target nucleus is greater than that of masses of the product nuclei. The K.E. of the product nuclei being greater than that of the incident particle. If Q is -*ve* the reaction is said to be endoergic, *i.e.,* energy must be supplied usually as a K.E. of the incident particle. A reaction cannot take place unless particles y and Y emerge with positive kinetic energies, *i.e.,*

$$E_y + E_Y \geq 0 \quad \text{or} \quad Q + E_x \geq 0$$

Although this condition is necessary, it is not sufficient.

The term E_Y in equation (2) represents the recoil energy of the product nucleus. It is usually small and hard to measure but can be eliminated by taking into account the conservation of momentum. In an experiment to measure a Q value, the bombarding energy E_x and the energy of the ejected particle E_y at some specified angle θ are measured. Thus by applying the laws of conservation of momentum, we have

$$M_x v_x = M_Y V_Y \cos \Phi + m_y v_y \cos\theta \quad \text{----------(5)}$$

$$M_Y V_Y \sin \Phi = m_y v_y \sin \theta \quad \text{----------------(6)}$$

Where v_x, v_y and V_Y are the velocities of incident particle, ejected particle and of product nucleus respectively. Eliminating Φ from equations (5) and (6), we have

$$M_Y^2 V_Y^2 = m_x^2 v_x^2 + m_Y^2 v_Y^2 - 2 m_x m_y v_x v_y \cos\theta$$

Since $E_x = ½\, m_x v_x^2$, $E_y = m_Y v_Y^2$ and $E_Y = ½\, M_Y V_Y^2$, hence after eliminating v_y, v_x and V_Y we get

$$2E_Y M_Y = 2E_x m_x + 2\, E_y m_y - 4(m_x m_y E_x E_y)^{1/2} \cos\theta,$$

$$E_Y = E_x \frac{m_x}{M_Y} + E_y \frac{m_y}{M_Y} - \frac{2}{M_Y} (m_x m_y E_x E_y)^{1/2} \cos\theta$$

Substituting the value of E_r in equation (3), we get

$$Q = E_y \left(1 + \frac{m_y}{M_Y} - \right) - E_x\left(1 - \frac{m_x}{M_Y}\right) - \frac{2}{M_Y} (m_x m_y E_x E_y)^{1/2} \cos\theta$$

This is known as Q equation. It gives the desired relation between the energy released and the measured quantities E_x, E_y and θ in lab-system. It is independent of the reaction mechanism and can be applied to all types of two body non-relativistic reaction processes. For a special case when we are observing the out coming particle y at 90^o to a collimated beam of projectile, the above relation reduces to

$$Q = E_y (1 + m_y / M_Y) - E_x ((1 - m_x / M_Y))$$

It can be utilized for the particular cases:

i) Elastic scattering, in which $m_x = m_y$ and $M_X = M_Y$; $Q = 0$

ii) Inelastic scattering, in which $m_x = m_y$ and $M_X = M_Y$ but

$$Q = -E^*,$$

Where E* is the excitation energy imparted to the target nucleus M_X

If E_Y^* is the excitation energy of the product nucleus Y, then the Q value of the equation is given by $Q_0 = E_y + E_Y - E_x + E_Y^*$.

The quantity $Q_0 - E_r^*$ is denoted as Q and is called Q – values, for nuclear reaction producing charged particles, to an accuracy of I part in a thousand or better. m_y / M_Y

Gamow's theory of Alpha Decay

An α-particle is emitted from a heavy nucleus as a tightly bound assembly of two neutrons and two protons from the pre-existed heavy nucleus.

In experiments, on the scattering of α – particles, it was found that, even the fastest of such particles from radioactive sources, having energy of 10 MeV, are repelled by atomic nuclei. The alpha particles have the repulsive forces due to the charges and some strong attractive nuclear short range forces. Due to the rapid decline of nuclear forces with distance, actively charged particle will experience diminishing attraction near the surface of nucleus when receding from the latter and at a certain distance is equal to the nuclear radius R, the forces of attraction will be balanced by the Coulomb force of repulsion. From this it follows that the internal part of the nucleus is separated from the outer space by a certain potential barrier, which prevents penetration of an α-particle into the nucleus.

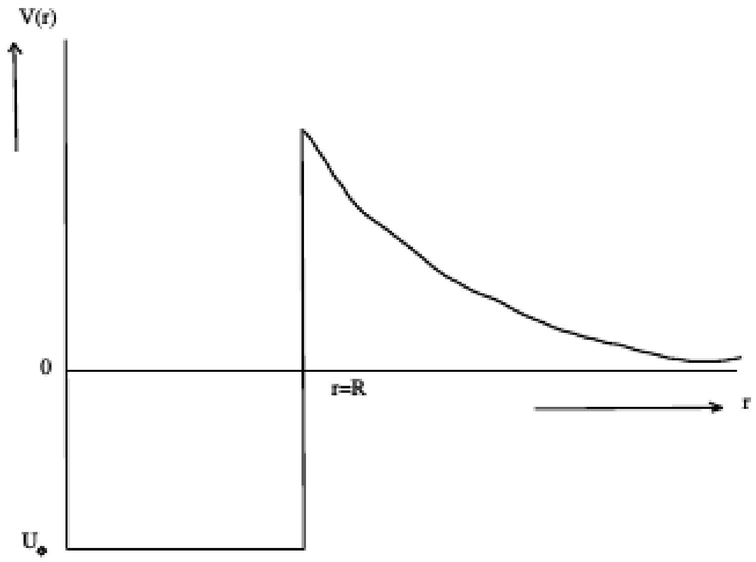


Figure : Potential energy curve

. The height of this barrier is the potential energy of an α particle at r = R. The potential energy V(r) of an alpha-particle outside the nucleus at a distance r from the centre of the nucleus is given by

$$V(r) = \frac{2(Z-2)e^2}{4\pi\varepsilon o r} \quad \text{for } r > R.$$

Where (Z-2) is the atomic number of the daughter nucleus. In the case of U^{238} the height of the potential barrier for an alpha particle is given by,

$$V(R) = \frac{2(Z-2)e^2}{4\pi\varepsilon o R}$$

$$= \frac{2x90x(1.6x10^{-19})^2}{10^{-14}} \text{ x } 8.99 \text{ x } 10^9$$

$$= 28\text{MeV}$$

The interaction in the nucleus may be represented by a constant attractive potential U_0 exerted over a distance R. Hence the potential energy.

V (r) = -Uo for r < R.

The coulomb potential and the constant potential energy Uo are joined, by a straight line at r = R.

If the motion of a particle in the neighborhood of a potential barrier is treated wave mechanically, it is found that there is a finite probability that the particle can leak through the barrier even though its kinetic energy is less than the height of the barrier. The probability that an α – particle can leak through the barrier can be calculated

Consider one dimensional Coulomb potential barrier of rectangular shape of width a, and height V, which is greater than the kinetic energy of an alpha particle. There are three regions. The Schrodinger equation in regions I and III is,

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}Eu = 0 \qquad \text{.............................(1)}$$

Where $m = \frac{M_\alpha M_D}{M_\alpha + M_D}$ the reduced mass of the alpha particle and the residual nucleus.

The equation in region II is,

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}(E - V) = 0 \qquad \text{…………………(2)}$$

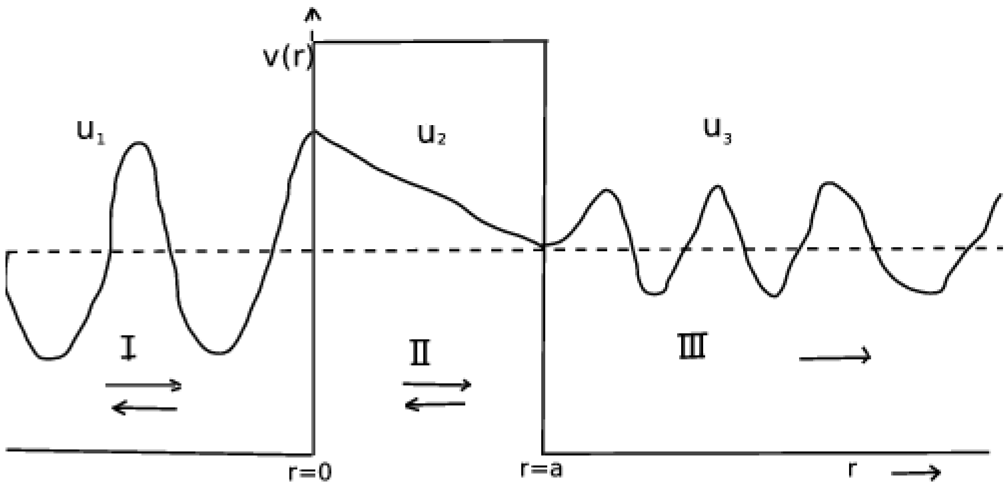


Figure : Tunnel effect

The region I has both incident and reflected alpha-waves. The solution of Equation (1) is

$$u_1 = A_1e^{ik1r} + B_1e^{-ik1r} \text{ …………….(3)}$$

Where $K_1 = \sqrt{2mE/\hbar}$

The region II has both forward moving transmitted wave and reflected wave from the other side of the barrier. The solution of equation (2) is

$$u_2 = A_2e^{k2r} + B_2e^{-k2r} \text{ ……………………….(4)}$$

Where $K_2 = \sqrt{2m(V-E)/\hbar}$

The region III has only forward moving transmitted wave. The solution of equation (1) is,

$$u_3 = A_3e^{ik1r} \text{ ……………….(5)}$$

The constants A_1, A_2, A_3, B_1 and B_2 are to be determined by using following boundary conditions.

$$u_1 = u_2 \quad \text{and} \quad \frac{\partial u_1}{\partial r} = \frac{\partial u_2}{\partial r} \quad \text{at } r=0$$

$$u_2 = u_3 \quad : \quad \frac{\partial u_2}{\partial r} = \frac{\partial u_3}{\partial r} \quad \text{at } r = a$$

By substituting the values of u_1, u_2 and u_3 in the above relations $u_1 = u_2$ at r=0

$$A_1 + B_1 = A_2 + B_2 \text{ …………… (6)}$$

$$\frac{\partial u_1}{\partial r} = \frac{\partial u_2}{\partial r} \quad \text{at } r = 0$$

$$ik_1A_1 - ik_1B_1 = k_2A_2 - k_2B_2 \text{ ……………..(7)}$$

$$u_2 = u_3 \quad \text{at } r = a$$

$$A_2e^{k2a} + B_2e^{-k2a} = A_3 e^{ik1a} \text{ ……………….(8)}$$

$$\frac{\partial u_2}{\partial r} = \frac{\partial u_3}{\partial r} \quad \text{at } r = a$$

$$A_2 k_2 e^{k2a} - B_2k_2e^{-k2a} = ik_1A_3 e^{ik1a} \text{ …………. (9)}$$

From equation 8 and 9

$$A_2 = ½ A_3 (1 + ik_1/k_2e^{(ik1 - k2)a} \text{ ………..(10)}$$

$B_2 = ½ A_3 (1 - ik_1/k_2 e^{(ik1 + k2)a}$ ………..(11)

From equation 6 and 7

$A_1 = ½ A_2 (1 + k_2 / ik_1) + ½ B_2 (1- k_2/ik_1)$ ………. 12

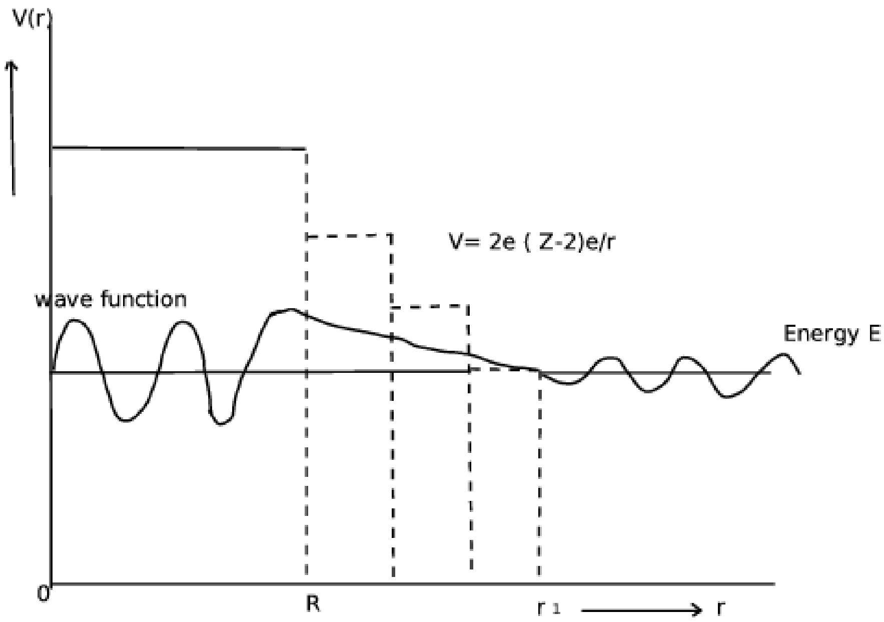


Figure : Mechanism of Alpha decay

Substituting equation 10 & 11 in equation 12

$A_1 = ¼ A_3 (1+ik_1/k_2) (1+k_2/ik_1)e^{(ik1 – k2)a} + ¼ A_3 (1- ik_1/k_2) (1- k_2/ik_1)e^{(ik1 – k2)a}$ …….(13)

As velocity of α – particle in I region is same as in III region. Hence the transmission probability of incident α – particle.

$$T = \frac{Transmitted \quad flux}{Incident \quad flux} = \frac{|A3|^2 v}{|A1|^2 v} = \frac{|A3|^2}{|A1|^2}$$ ………(14)

In practice $k_2a >> 1$, hence first term of equation 13, can be neglected in comparison to the second.

Therefore $A_1 = ¼ A_3 (1-ik_1/k_2) (1-k_2/ik_1)e^{(ik1 + k2) a}$

$A_1/A_3= ¼ (1-ik_1/k_2) (1-k_2/ik_1)e^{(ik1 + k2) a}$

$$\frac{|A1|^2}{|A3|^2} = \frac{A1}{A3} x \left[\frac{A1}{A3}\right]^* = \{\ \frac{1}{4}\ (1-ik_1/k_2)\ (1-k_2/ik_1)e^{(ik1\ +\ k2)\ a}\ \}\{\ \frac{1}{4}\ (\ 1+ik_1/k_2)\ (1+k_2/ik_1)e^{(-ik1\ +\ k2)\ a}\ \}$$

$$= \frac{1}{16}\left(1-\frac{ik_1}{k_2}\right)\left(1+\frac{ik_1}{k_2}\right)\left(1-\frac{k_2}{ik_1}\right)\left(1+\frac{k_2}{ik_1}\right)e^{2k_2a}$$

$$= \frac{(k_1{}^2 + k_2{}^2)^2}{16k_1{}^2k_2{}^2} e^{+2k_2a}$$

Transitivity of the barrier $T = \frac{(k_1{}^2 + k_2{}^2)^2}{16k_1{}^2k_2{}^2} e^{-2k_2a}$ ………..(15)

When $2k_2 >> 1$, the most important factor in this equation is the exponential term

$T = e^{-2k_2a}$ ....(16)

This is the Gamow's Formula.

This equation represents the fraction of the α –particles that will penetrate the barrier of width a and hight V(>E). If the potential is not constant in the region o< r < a, we can approximate it with a series of small steps. The total probability is the product of individual probabilities.

$T = e^{-2 \int k2\, dr}$

T = exp [-2 √2m/ħ ∫ (V(r) – E)½ dr

The integral is taken through the whole region between R and r_1.

Let us assume that an alpha particle moves inside the potential well with velocity v_o and hence hits the wall $u_o/2R$ times per second. Multiplication of frequency with escape probability T will give us decay constant. (disintegration constant λ)

λ = v /2R x T

$$\lambda = \frac{v_0}{2R} \exp\left[\frac{-2\sqrt{2m}}{\hbar}\right] \int_R^{r1} (V(r) - E)^{(1/2)} dr$$

Taking logarithm

$$\log_e \lambda = \log_e \frac{\nu_0}{2R} \exp\left[\frac{-2\sqrt{2m}}{\hbar}\right] \int_R^{r1} (V(r) - E)^{(1/2)} dr$$

$$\log_e \lambda = \log_e \frac{\nu_0}{2R} - \left[\frac{-2\sqrt{2mE}}{\hbar}\right] \int_R^{r1} \frac{2(Z-2)e^2}{4\pi\varepsilon_0 rE} - 1)^{\frac{1}{2}} dr$$

Hence upper limit of the integral $r_1 = 2(Z-2)\ e^2/4\pi\varepsilon_0 E$. The integral may be simply determined by means of substitution

$r = r_1 \cos^2 \psi$. and $R = r_1 \cos^2 \psi_0$

$$\log_e \lambda = \log \frac{vo}{2R} + \frac{4\sqrt{2mE}}{\hbar} \int_{\psi_0}^{0} \sin^2 \psi d\psi$$

$$= \log_e \frac{vo}{2R} + 2\frac{\sqrt{2mE}}{\hbar}{}_1 \left[-\psi_0 + \sin\psi_0 \cos\psi_0\right]$$

$$\log_e \lambda = \log \frac{vo}{2R} + \frac{2\sqrt{2mE}}{\hbar}{}_1 \left[\cos^{-1} \frac{R}{r_1}\right]^{\frac{1}{2}} \left[\left(\frac{R}{r_1}\right)^{\frac{1}{2}} \left(1 - \frac{R}{r_1}\right)^{\frac{1}{2}}\right]$$

since $R \ll r_1$, hence

$$\cos^{-1}\left(\frac{R}{r_1}\right)^{\frac{1}{2}} \approx \frac{1}{2}\left(\frac{R}{r_1}\right)^{\frac{1}{2}}$$

$$1 - \left(\frac{R}{r_1}\right)^{\frac{1}{2}} \approx 1$$

$$\log_e \lambda = \log_e \frac{vo}{2R} - \frac{2\sqrt{2mE}}{\hbar} r_1 \left[\frac{\pi}{2} - 2\left(\frac{R}{r_1}\right)^{\frac{1}{2}}\right]$$

$$= \log_e \frac{vo}{2R} + \frac{4e}{\hbar}\left(\frac{m}{\pi\varepsilon 0}\right)^{\frac{1}{2}} (Z-2)^{\frac{1}{2}} R^{\frac{1}{2}} - \frac{e_2}{\hbar\varepsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}} (Z-2)^{\frac{1}{2}}$$

–

$$\log_e \lambda = \log_e \frac{vo}{2R} + 2.97 Z_D{}^{1/2} R^{1/2} - 3.95 Z_D E^{-1/2}$$

To compare the theory with observations more easily, we may take logarithms to base 10 as.

$$\log_{10}\lambda = 20.46 + \log_{10}\left(E^{\frac{1}{2}} A^{-1/3} \right) - 1.72 Z_D \ \mathrm{E}^{-1/2} + 1.42\left(Z_D A^{\frac{1}{3}} \right)^{\frac{1}{2}} \quad \text{.........(17)}$$

The changes in atomic number and nuclear radius are negligible when compared to changes in energy.

$$\log_{10}\lambda = a + bE^{-\frac{1}{2}} \quad \text{.......................(18)}$$

Where a and b are constants .

Equation 18 shows that the emitters having lesser decay constants emit alpha particles of greater energy E, which is Geiger and Nuttal law. This shows that the quantum mechanical theory of barrier penetration is able to account well for α decay.

Beta Decay

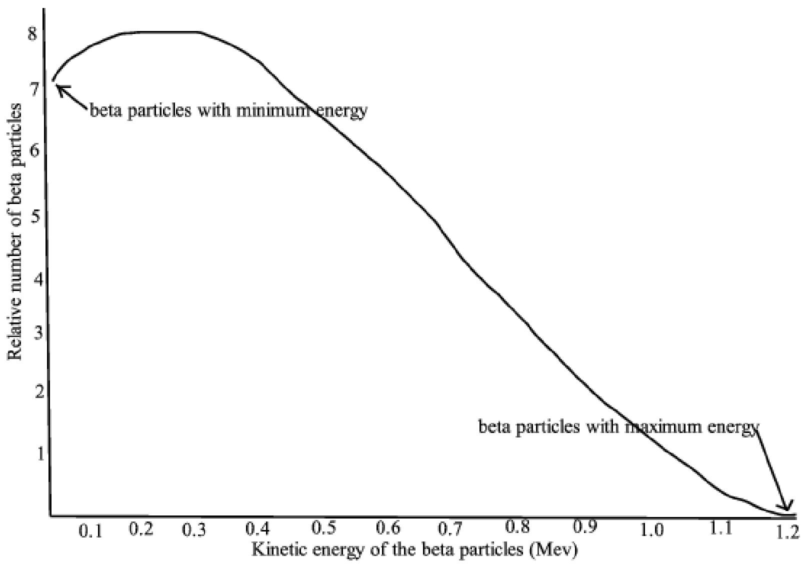


Figure : Beta ray spectrum of RaE

The energies of beta ray emitted from the radioactive substances are measured by means of beta ray spectrometer. The typical beta ray spectrum of Radium is shown in the figure. This type of spectrum shows that the beta rays have maximum energy below which there is continuous spectrum with average energy usually less than the half maximum. Every continuous beta spectrum has a definite maximum and the height and position of which depend on the nucleus emitting the particles. There is also a definite upper limit or End point of energy(minimum energy) beta particles emitted by the nucleus which is different for different beta emitting nuclides. There is an apparent failure to conserve the linear and angular momentum in beta decay.. In the process $^{14}_{6}C \rightarrow ^{14}_{7}H + ^{0}_{-1}e$ ------------- (1)

The nuclear angular momentum of ^{14}C and ^{14}N are found to be 0 and 1 respectively. The electron has intrinsic spin ½ therefore the angular momentum cannot be conserved during the transition. All these difficulties were eliminated by Pauli introducing the existence of a new hypothetical particles called neutrino. It has no charge, negligible mass and ½ spin. In beta emission process the decayed energy shared between beta particle and the neutrino . Thus conserving all the physical parameters.

$$^{14}_{6}C \rightarrow ^{14}_{7}H + ^{0}_{-1}e + \nu \quad \text{---------}(2)$$

Fermi theory of Beta Decay

In 1934, Fermi made a successful theory of beta decay based on Pauli's neutrino hypothesis. This theory is based on the following assumptions

1. The light particles, the electron and neutrino are created by the transformation of a neutron into proton in a nucleus.
2. The energy remains conserved in the decay process, the available energy being shared among the electron and the neutrino.
3. The neutrino has rest mass zero or very small compared to that of the electron.
4. The beta decay process is analogous to the emission of electromagnetic radiation by an atom with the electron neutrino field acting in the place of the electromagnetic field.
5. Time dependent perturbation theory is a very good approximation, because of the smallness of coupling constants.
6. Electro-neutrino field is weak.

7. No nuclear parity change occurs and higher order terms in R/λ can be neglected.

8. As nucleons move with velocities of only ≈ c /10 in nuclei.

Using time dependent theory and Dirac's expression, the probability that an electron of momentum between Pe and Pe+dPe is emitted per unit time may be written as

$$P(P_e)dP_e = \frac{2\pi}{\hbar}|Hif|^2 \frac{dN}{dE_o} \qquad \text{.............................(1)}$$

$\frac{dN}{dE_o}$ no.of quantum mechanical states of the final system per unit energy interval.

Hif → matrix element of the interaction for the initial and final states.

Interaction matrix element, Hif = $\int \psi_f^* H \psi_i d\tau$ .... (2)

ψ_f and ψ_i → wave functions of the system in its final state and in its initial state.

H→ Hamiltonian operator

dτ → volume element

ψ_i = ψ (parent nucleus) = ψ_i

ψ_f = ψ (daughter nucleus) ψ (electron) ψ (antineutrino)

= $\psi_f \psi_e \psi_{\bar{\nu}}$

Fermi suggested a new constant called fermi coupling constant denoted by, g.

using equation (2) ⇒ The matrix element becomes

$$H_{if} = g\int(\psi_f{}^*\psi_e{}^*\psi_{\bar{\nu}}^*)M\psi_i d\tau \qquad \text{................................ (3)}$$

= $g \int [\psi_f^* \psi_e^* \psi_\nu] M \psi_i d\tau$ [since the complex conjugate of $\bar{\nu} = \nu$]

M→ dimensionless matrix elements

ψ_ν = $V^{-½} \exp [-(i/\hbar) P_\nu .r]$

ψ_e^* = $V^{-½} \exp [-(i/\hbar) P_e.r]$

P_v & $P_e \rightarrow$ momenta of the neutrino & electron

r $\rightarrow$ position co-ordinate

Thus the matrix element becomes

$$Hif = g \int [\psi_f^* \left(\frac{1}{V^{12}} \exp(-i/\hbar) P_v.r\right) \left(\frac{1}{V^{1/2}} \exp(-i/\hbar) P_e.r\right)] M\psi_i \, d\tau$$

Equation (3) $\Rightarrow Hif = g \int [\psi_f^* \frac{1}{V} \exp(-i/\hbar) P_e + P_v).r] M\psi i d\tau$ ....(4)

The exponential term can be written as

$$\exp\left[\frac{-i}{\hbar}(P_e + P_v)r\right] 1 - \frac{i}{\hbar}(P_e + P_v).r - \frac{1}{2\hbar^2}[(P_e + P_v).r]^2 + \ldots$$

Here r is no greater than the nuclear radius R. Pe & Pv are both order of magnitude mc,

$$2mc\, R/\hbar \approx 1/50$$

The matrix element can be written as

$$Hif = \frac{g}{V} \int \psi_f M \, \psi_i d\tau$$

$$= \frac{g}{V} |Mif| \qquad \text{..............................(5)}$$

Mif $\rightarrow$ nuclear matrix element of the final and initial wave functions of the nucleus.

To find the Statistical factor (final state Density) dN/dE_o

The position and momentum of e^{-n} or neutrino can be represented by a point in phase space, the space containing three spatial and the three momentum dimensions

$\Delta x. \Delta y. \Delta z. \Delta Px. \Delta Py. \Delta Pz \approx h^3$

The number of states of a particle restricted to a volume V in actual space and whose momentum lies between the limits P and P+dP is given by

$dN = V \times 4 \pi P^2 dP / h^3$ …………………...............(6)

For the electron, $dN_e = 4\pi V P_e^2 dP_e / h^3$

neutrino, $dN_v = 4\pi V P_v^2 dP_v / h^3$

As electron and neutrino are independent of one another, hence the no.of states available to them jointly is

$dN = dN_e \cdot dN_v$

From equation (6) $[\,dN\ (4\pi V P_e^2 dP_e]\ /h^3)\ \ x\ (\ [\ 4\pi V P_v^2 dP_v\ ]\ /h^3)$

$= [\ 16\pi^2 V^2 P_e^2 P_v^2 dP_e\, dP_v / h^6\ ]$

The no.of states per unit energy of the electron is,

$$\frac{dN}{dE_o} = \frac{16\pi^2 V^2}{h^6} Pe^2\, Pv^2\, dPe \frac{dPv}{dE_o} \qquad \text{.......................................(6)}$$

The total available energy

$E_o = E_v + E_e$ ....(7)

For fixed electron energy Ee,

$dE_o = dE_v$ .... (8)

The momenta Pe and P_v are related to the electron and neutrino energy respectively by the equations

$E_v = C\, P_v$ ....(9)

$dE_v = C.\, dP_v$ ....(10)

Using eqns (7), (8), (9) and (10) in eqn (6), we get

$$\frac{dN}{dE_o} = \frac{16\pi^2 V^2}{h^6} Pe^2 \left(\frac{E_o - E_e}{c}\right)^2 dPe.\frac{1}{c}$$

Inserting statistical factor and Hif into eqn (1), we get the probability

$$P(Pe)dPe = \frac{2\pi}{\hbar}|Hif|^2 \frac{dN}{dE_0} \qquad [\hbar = h/2\pi\ ;\ \text{and}\ \ h = \hbar.2\pi\ ]$$

$$= \frac{2\pi}{\hbar}\frac{g^2}{V^2}|Mif|^2 \frac{16\pi^2 V^2 Pe^2}{h^6}\left(\frac{E_0 - E_e}{c}\right)^2 dPe.\frac{1}{c}$$

$$= \frac{32\pi^3 g^2}{\hbar.\hbar^6.(2\pi)^6}|Mif|^2 \; Pe^2 \; \frac{(E_0 - E_e)^2}{c^3} dPe$$

$$= \frac{32\pi^3 g^2}{\hbar^7 \, 64\pi^6 c^3}|Mif|^2 \; Pe^2 (E_o - E_e)^2 dPe$$

$$P(Pe)dPe = \frac{g^2|Mif|^2}{2\pi^3 c^3 \hbar^7}(E_o - E_e)^2 \; Pe^2 \; dPe$$

Fermi and Gamow Teller Selection Rules

The β-decay life-times fell into two categories, accordingly transitions , and are allowed and forbidden. These transitions are governed by certain selection rules. This is applicable when the electron and neutrino are emitted with their intrinsic spins antiparallel (Singlet State), the change in nuclear spin Δ*J* must be strictly zero, if these are emitted with their spins parallel (*triplet state*), ΔJ may be + 1, 0, or -1, (but no $J_i = 0$ to $J_f = 0$), where subscripts i and f refer to the initial and final nuclear states. The former selection rule was one originally proposed by Fermi, the latter was subsequently suggested by Gamow and Teller. In both types of allowed transitions orbital angular momentum and parity are left unchanged. The interactions that give rise to Fermi and Gamow Teller (*G-T)* selection rules are different. Experiment shows that the allowed transitions of the type ΔJ = 1, obeying *G - T* selection rule, are forbidden by Fermi-selection rule, as in the decay

$^{4}_{2}He \rightarrow ^{6}_{3}Li + \beta^- + \bar{v}$ ($0^+ \rightarrow 1^+$) log ft = 2.77

$^{14}_{6}C \rightarrow ^{14}_{7}N + \beta^- + \bar{v}$ ($0^+ \rightarrow 1^+$) log ft =2.62

$^{75}_{32}Ge \rightarrow ^{75}_{33}As + \beta^- + \bar{v}$ ($1/2^- \rightarrow 3/2^-$) log ft =5

There are also allowed transitions of the 0 →0 type that are allowed by Fermi selection rules but forbidden by G - T selection rules, e.g.,

$^{14}_{8}O \rightarrow ^{14}_{7}N + \beta^- + v$ ($0^+ \rightarrow 0^+$) log ft =3.487

$^{26}_{13}Al \rightarrow ^{26}_{12}Mg + \beta^{+} + v$ $(0^{+} \rightarrow 0^{+})$ log ft = 3.48

$^{34}_{17}Cl \rightarrow ^{34}_{16}S + \beta^{+} + v.$ $(0^{+} \rightarrow 0^{+})$ log ft =3.49

However, many transitions are allowed by both selection rules. This is always possible in the allowed decays in which $J_i = J_f \neq 0$. The examples are

$^{1}_{0}n \rightarrow p + \beta^{+} + \bar{v}$ $(1/2^{+} \rightarrow 1/2^{+})$ log ft =3.074

$^{3}_{1}H \rightarrow ^{3}_{2}He + \beta^{+} + \bar{v}$ $(1/2^{+} \rightarrow 1/2^{+})$ log ft =3.05

$^{35}_{16}S \rightarrow ^{35}_{17}Cl + \beta^{-} + \bar{v}$ $(3/2^{+} \rightarrow 3/2^{+})$ log ft =5.0

$^{17}_{9}F \rightarrow ^{17}_{8}O + \beta^{+} + v$ $(5/2^{+} \rightarrow 5/2^{+})$ log ft =3.4

$^{7}_{4}Be \rightarrow ^{7}_{3}Li + \beta^{+} + v$ $(3/2^{+} \rightarrow 3/2^{+})$ log ft =3.36

These allowed transitions are further classified as favoured (*super allowed*) and unfavoured (*allowed*) Transitions. The allowed transition is said to be favoured if the nucleon which changes its charge remains in the same level, it is unfavoured if the nucleon changes its level. Most of the allowed β-transitions are unfavoured. For the β^{-} decay, the change of a neutron into a proton without the change of level would increase the total energy of the nucleus and would, therefore, not lead to a spontaneous decay. For the β^{+} -decay, the surplus of neutrons make the allowed transitions unfavoured. There are few exceptions,

Example: $^{3}_{1}H \rightarrow ^{3}_{2}He$; $^{6}_{2}He \rightarrow ^{6}_{3}Li$

The conditions for super allowed transitions are same as that for unfavoured or allowed transitions. The matrix element is also energy independent. The main difference between the two cases is that the unfavoured or simply allowed transitions are not between mirror nuclei.

Table : Nuclear decay in mirror nuclei

Nuclear Decay	*Transition*	$T_{1/2}$	E_{max} *MeV*	Log ft
${}^{11}_{6}C \to {}^{11}_{5}B + \beta +$	$3/2^- \to 3/2^-$	1230	0.99	3.6
${}^{19}_{10}Ne \to {}^{19}_{9}F + \beta +$	$1/2^+ \to 1/2^+$	20.3	2.18	3.4
${}^{27}_{14}Si \to {}^{27}_{13}Al + \beta +$	$5/2^+ \to 5/2^+$	4.9	3.64	3.7

Let us now consider what happens when the transition from initial to final nucleus does not take place by the emission of *S*-wave electron and neutrino. Because of the finite size of the nucleus, the electron and neutrino emission with orbital angular momenta other than zero is also possible. The magnitudes of the wave functions Ψ_e & Ψ_v for *p*-wave, *d*-wave, etc., over the nuclear volume decrease rapidly with increasing orbital angular momentum. Beta transitions with angular momentum, carried off by the two light particles together. $l_\beta = 1, 2, 3$ etc., are classified as first, second, third etc., *forbidden transitions*.

If l_β is odd, initial and final nuclei must have opposite parities (*parity changes in these transitions*); for even l_β values the initial and final nuclei must have same parity (*no change in parity*). Furthermore, as in allowed transitions, the emission of leptons (electron and neutrino) in the singlet state (Fermi-selection rule) require $\Delta J \leq l_\beta$, whereas triplet-state (*G-T* selection rule) emission requires $\Delta J \leq l_\beta + 1$. Thus selection rules for forbidden transitions are

First forbidden – For these transitions $l_\beta = 1$ and parity changes.

Fermi – selection rules: $\Delta J = \pm 1, 0$ (except $0 \to 0$).

Gamow Teller rules: $\Delta J = \pm 2, \pm 1, 0$ (except $0 \to 0$, $½ \to ½$, $0 \leftrightarrow 1$)

Internal Conversion

The nucleus in an excited state can perform a transition to a lower state not only by emitting a light quantum but also by transmitting energy directly to the electrons surrounding the nucleus. The transition to lower state is then connected with the ejection of an atomic electron, called a conversion electron, from a bound orbit. The kinetic energy of the rejected electron is equal to this transition energy ($E_i - E_f$) minus the binding energy E_B of the orbital electron. As the gamma ray is internally converted into electron, the process is known is internal conversion. These electrons produce a series of mono energetic lines, and not a continuous spectrum as in β^- decays.

The line with the lowest energy that of K shell electron, $E_{eK} = (E_i - E_f) - E_K$. This is followed by the line corresponding to the conversion of L electrons. $E_{eL} = (E_i - E_f) - E_L$, etc…. The transition energy can be measured accurately from the spectrum of the conversion electrons.

It is similar to the atomic Auger effect, instead of emitting a photon when as atomic electron jumps from a higher to a lower energy orbit, one of the atomic electrons is ejected. The gamma emission is caused by transverse electric and magnetic fields. The internal conversion is produced by the time varying Coulomb field of the nucleus, which has a radial direction. Thus the process of internal conversion is the conversion of nuclear excitation energy to kinetic energy through the direct electromagnetic interaction between the electrons and nucleons without any intermediate γ-rays. For all electromagnetic transitions, except the zero-to-zero transition, gamma ray emission and internal conversion compete.

The possible modes of decay excluded the total transition probability λ from a nuclear state a to a nuclear state b is the sum of two terms.

$$\lambda = \lambda_e + \lambda_\gamma,$$

Where λ_e and λ_γ are the partial decay constants for conversion electron emission and for gamma emission respectively. The ratio between these two decay constants is called the conversion coefficient and is measured as the ratio between the total number of conversion electrons emitted over a given time divided by the number of gamma rays emitted in the same transition over the same time.

Conversion coefficient

$$\alpha = N_e/N_\gamma = \lambda_e / \lambda_\gamma.$$

$$\lambda = \lambda_\gamma (1 + \alpha),$$

α may have any value between 0 and ∞. Since the probability of decay per unit time λ is related with the width Γ as $\lambda = \Gamma / \hbar$

$$\Gamma = \Gamma_e + \Gamma_\gamma \text{ and } \alpha = \Gamma_e / \Gamma_\gamma$$

the radiative width Γ_γ = Total width $\Gamma / 1 + \alpha$)

in terms of means lives $\tau = \tau_\gamma / (1 + \alpha)$.

As a result of internal conversion, for each gamma ray there may be several conversion lines corresponding to the ejection of electrons from different atomic shells, K, L_1, L_{11}, L_{111}, M_1 ….. therefore, total conversion co-efficient is given by

$$\alpha = \frac{N_K + N_L + N_M + \dots}{N_\gamma} = \frac{N_K}{N_\gamma} + \frac{N_L}{N_\gamma} + \frac{N_M}{N_\gamma} + \dots..$$

$$= \alpha_K + \alpha_L + \alpha_M = \sum_{k=K,L,M,\dots} \alpha_k$$

Here $\alpha_K, \alpha_L, \alpha_M$ … etc., are the partial conversion coefficients.

Nuclear Isomerism

The delayed transitions are called isomeric transitions and the states from which they originate are called isomeric states or isomeric levels. Nuclear species which have the same atomic and mass number, but have different radioactive properties, are called nuclear isomers and their existence is referred to as nuclear isomerism. Nuclides that are isomeric states of a given isotope differ from each other in energy and in angular momentum.

After the discovery of artificial radioactivity, indications came from several different directions that other nuclides exist in isomeric forms. When a sample containing bromine was bombarded with slow neutrons, the product was found to show three different half lives for beta decay : 18 min, 4.5 hr and 34 hr. Chemical tests showed that the radioactive elements were isotopes of bromine. This result was surprising because the reactions with slow neutrons are invariably of the (n, γ) type and since ordinary bromine consists of two isotopes only, Br^{79} and Br^{81} not more than two radioactive products ^{80}Br and ^{82}Br [$^{79}Br\ (n,\gamma)\ ^{80}Br$ and $^{81}Br\ (n,\gamma)\ ^{82}Br$] were to be expected. When bromine was bombarded with 17 MeV gamma ray two products ^{78}Br and ^{80}Br [$Br^{79}\ (\gamma,n)\ Br^{78}$, $Br^{81}\ (\gamma, n)\ Br^{80}$], with three decay periods, 6.4 min, 18 min, 4.4hr were obtained. Two of these periods (4.4 hr and 18 min) are common to both sets of reactions and must, therefore, be assigned to the isotope that is common to both sets of reactions namely, ^{80}Br.

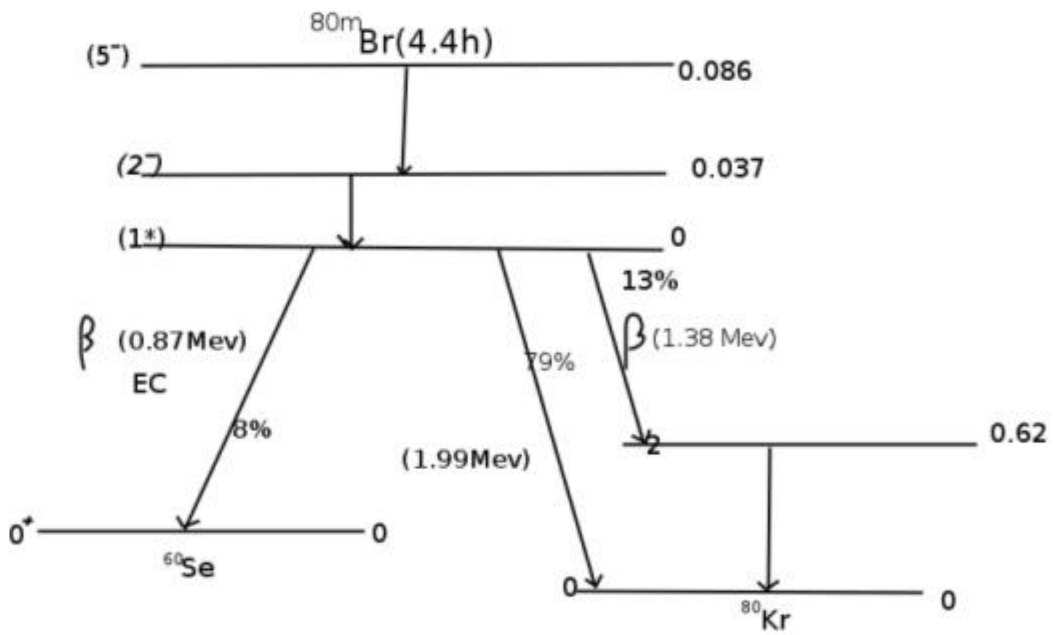


Figure : Decay scheme of nuclear isomer ^{80}Br

The two half lives were attributed to two isomeric states of ^{80}Br. The difference between the nuclear isomers is attributed to a difference of nuclear energy states, one isomer represents the nucleus in its ground states, of ^{80}Br. The difference between the nuclear isomers is attributed to a difference of nuclear energy states, one isomer represents the nucleus in its ground state, whereas the other is the same nucleus in an excited state of higher energy, or the metastable state. Most known γ-decay rates have been determined by the direct measurement of the life-times of the exited states.

Total decay rate $\lambda = \lambda_\gamma + \lambda_e = \lambda_\gamma (1 + \alpha)$

$T_{1/2} = (\log_e 2)/ \lambda = 0.693\ \tau_\gamma\ / (1 + \alpha)$.

Since the internal conversion coefficient α can be measured or can be calculated theoretically and half life $T_{½}$ can be measured, hence τ_γ the average life or λ_γ the rate of photon emission can be calculated.

UNIT III : NUCLEAR FORCES AND PROPERTIES OF NUCLEAR FORCES

Deuterons – properties of deuteron- ground state of deuteron – excited state – magnetic quadrupole moment of deuteron- neutron-proton scattering at low energies – proton–proton scattering at low energies – meson theory of nuclear forces- reciprocity theorem – Breit-Wigner one level formula

Deuteron

One of the main objectives of the study of atomic nuclei is to understand the basic nature of the forces that bind nucleons together. A number of characteristics of nuclear forces and of nuclear structure have been established.

The protons and neutrons are very strongly bound within the nucleus. The nature of the force, which binds them together is basically different from the more familiar types of forces e.g. the gravitational or the electromagnetic forces. The gravitational force is far too weak to account for the nuclear binding. For instance the potential energy of gravitational interaction between two nucleons within the nucleus at a distance of 2×10^{-15} m from one another energy is 5.75×10^{-32} MeV.

This is much smaller than the binding energy per nucleon, which is of the order of a few million electron volts. So far as the electromagnetic force is concerned two protons repel one another due to like charges on them. Again the neutrons being electrically neutral cannot have any electromagnetic interaction between themselves or with the protons.

Thus we must assume a type of force other than the above two act between the nucleons within the nucleus. This force is very strongly attractive up to a certain maximum distance between the nucleons which is of the order of about 2 fm. This distance is known as the range of the force. Beyond that distance the force is negligibly small. It is known as the strong interaction. The exact nature of this force is not fully understood. However, some idea of its nature can be inferred from the ground stable properties of the deuteron the simplest proton-neutron bound system.

Properties of Deuteron

1) The extraordinary stability of the alpha particle shows that the most stable nuclei are those in which number of neutrons and protons are equal. The deuteron consists of two particles of roughly equal masses M, so that the reduced mass of the system is ½ M.

2) The binding energy of the deuteron is very small. Its experimental value is 2.2 MeV. Since the energy needed to pull a nucleon out of a medium mass nucleus is about 8 MeV, we must regard the deuteron as loosely bound.

3) The angular momentum quantum number, often called the nuclear spin, of the ground state of the deuteron determined by a number of optical, radiofrequency and micro-wave methods is one. It suggests that the spins are parallel (triplet state) and the orbital angular momentum of the deuteron about their common center of mass is zero. Thus the *ground state is* ^3S *state*.

4) The parity of deuteron as measured, indirectly, by studies of nuclear disintegrations and reactions for which certain rules of parity changes exist, *is even*

5) The sum of the magnetic dipole moments of the proton ($2.79\mu_N$) and neutron ($-1.9\ \mu_N$), does not exactly equal to magnetic moment of the deuteron ($0.857405\ \mu_N$), measured by magnetic resonance absorption method.

6) A radiofrequency molecular beam method has been employed to determine the quadrupole moment of the deuteron as $Q = +\ 0.00285 \times 10^{-28}\ cm^2$. This shows the departure from spherical symmetry of a charge distribution. The +ve sign indicates that this distribution is prolate rather than oblate. The electric quadrupole moment and the magnetic moment discrepancy can be explained if the ground state is a mixture of the triplet states $^3S_1$ and $^3D_1$ having even parity. The percentage probability of finding the deuteron in D-state is 4 ± 2%. As deuteron spends most of the time in the spherically symmetrical state.

7) Since the neutron has no charge, the force between the neutron and proton cannot be electrical. This force cannot be magnetic as magnetic moments are very small. It cannot be gravitational force, as the masses are very small. So we must accept the nuclear force as a new type of force. This force is short range, attractive and along the line joining the two particles (*central force*). Since a central force cannot account for the quadrupole moment of the deuteron.

8) The force depends only on the separation of the nucleons not on the relative velocity or orientation of the nucleon spins with respect to the line. This force can be derived from a potential. Since the force is attractive, $V(r)$ is negative and decreases with decreasing r. Since it is short range, $V(r)$ vanishes for $r > b$, where $b \sim 3$ fermi.

Ground state of the Deuteron

The Schrodinger equation for a two body problem is $\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$ ---(1)

M is the reduced mass and E is the energy of the system equal to the binding energy of deuteron and V the potential energy describing the forces acting between the two bodies.

The Schrodinger equation for the ^3S state (l=0) of deuteron is

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d\psi(r)}{dr}\right]+\frac{2m}{\hbar^2}(E-V(r))\psi(r)=0 \text{-----------------(2)}$$

In this case the reduced mass m=1/2M where M is the mass of the nucleon. We expect the ground state to be spherically symmetrical(S state) so that ψ(r) depend only on r (central modified force. Substituting ψ(r) =u(r) /r in the above equation , where u(r) is called the radial wave function

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}(E-V(r))u=0 \text{ ----------------(3)}$$

The wave function of the bound state of deuteron is not markedly depended on the exact shape of the potential V(r) between the proton and neutron provided the potential of short range is chosen.

For the ground state of deuteron the total energy E is negative and equal to –B, where B is the binding energy of the deuteron. Thus equation 3 can be written as

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}(V_0-B)u=0 \qquad \text{for } r < b \text{ ----------------------(4)}$$

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}(-B)u=0 \qquad \text{for } r > b \text{ ----------------------(5)}$$

These equation can be written as

$$\frac{d^2u}{dr^2}+K^2u=0 \qquad \text{for } r < b \text{ --------(6)}$$

$$\frac{d^2u}{dr^2} + \alpha^2 u = 0 \qquad \text{for } r > b \text{--------------- (7)}$$

Where $K^2 = M(V_0 - B)/\hbar^2$ and $\alpha^2 = MB/\hbar^2$

The general solution for equation 6 and 7 are

$$u = A_1 SinKr + B_1 \qquad \text{for } r < b \text{ ----------------(8)}$$

$$u = A_2 e^{\alpha r} + B_2 e^{-\alpha r} \qquad \text{for } r > b \text{ -----------------(9)}$$

The boundary conditions are at $u(r \to 0) = 0$ to keep wave function ψ finite.

$u(r \to \infty) = 0$, u must not diverge faster than as $r \to \infty$

To satisfy the conditions at zero and infinity the solution reduces to

$$u = A_1 SinKr \qquad \text{for } r < b \text{ -------------------(10)}$$

$$u = B_2^{-\alpha r} \qquad \text{for } r > b \text{ ----------------------(11)}$$

We can get the values of two constants A_1 and B_2

$$A_1 = \frac{1}{2\pi}\left[\frac{\alpha}{\alpha b + 1}\right]^{\frac{1}{2}} \text{ ---------------------------(12)}$$

$$B_2 = A_1 SinKbe^{\alpha b} = \left[\frac{\alpha}{2\pi(\alpha b + 1)}\right]^{\frac{1}{2}} \frac{K}{K_0}^{\alpha b} \text{ ----------------(13)}$$

On substituting the value of α=0.232 fm $^{-1}$ and b= 1.93 fm in the above equations

We get A_1 = 0.16 and B_2 = 0.247

With these values it was inferred that nucleons in the deuteron spend only one third of the time with in the range of nuclear force and thus deuteron is loosely bound.

Excited state of the deuteron

First write the radial part of the Schrodinger equation for any angular momentum ι

$$\frac{d^2u_1(r)}{dr^2}+\left[K^2-\frac{l(l+1)}{r}\right] \quad \text{for } r \le b \quad \text{---------------(14)}$$

$$\frac{d^2u_1(r)}{dr^2}+\left[\alpha^2-\frac{l(l+1)}{r}\right] \quad \text{for } r \ge b \quad \text{-------------(15)}$$

Where K^2 and α^2 are having their usual values .

The general solution of the equation 14 involves spherical Bessel function j_l and spherical Neumann function n_l. As n_l approaches $-\infty$ as r → 0 , thus the solution of equation 14 is

$$u_1 = Aj_l(Kr) \quad \text{------------------ (16)}$$

Where $j_l(Kr) = (\pi/2Kr)^{1/2} J_{l+1/2}(kr)$ where $J_{l+1/2}$ being the function of half odd integer order

The solution of equation 15 outside the range of nuclear force

$$u_1(r) = B[j_l(i\alpha r) + in_l(i\alpha r)]$$

for r>b --------------------(17)

Using the boundary conditions that the function and its first derivatives are continues at the edge of the well.

For b< 1.43 x 10^{-15}m , αb<1 and since α<<K the expression is very small and nearly zero. Thus

$$J_{l-1}(Kb) = 0$$

This condition holds for all angular momenta except *l*=0

Thus the minimum well depth is $V_0 = \frac{\pi^2\hbar^2}{Mb^2}$

If we choose $b=2x10^{-15}$ m; we get V_0 =144 MeV, which is almost four times as large as the actual well depth in the ground state. Repeating this procedure for larger and larger values of *l* we find that a deeper well depth is required to produce a bound state. Thus we conclude that no bound state exists for $l > 0$ for deuterons.

Experimentally determined properties

Deuteron is the only two nucleon bound system made up of a proton and a neutron. The two other possible two-nucleon systems, the diproton (^2He) and the dineutron, do not exist as bound system.

The following are the experimentally determined properties of the ground state deuteron.

(i)The binding energy is small E_{BE} =2.2245±0.0002 MeV.

The binding energy per nucleon in the deuteron is thus f_{BE}=1.1122MeV. This is much smaller than the mean value of the binding fraction (f_B=B/A) for the nuclei with mass numbers of 4 or more. Even for the α-particle with A=4, f_B=7.07 MeV. Thus it is clear that the deuteron is a rather weakly bound structure, compared to most other nuclei.

(ii) The spin of the deuteron (total angular momentum) in the ground state in the unit of ħ is J_d=1.; μ_d=0.857414±0.000019.

(iii)The deuteron also possess a small but finite electric quadrupole moment which has the value ; $Q_d = +0.282\times10^{-31}\ m^2$.

(iv) The parity of the deuteron ground state is even.

The observed values of the ground state spin and magnetic moment of the deuteron yield important information about the nature of this state.

The deuteron is made up of a proton and a neutron both of which are spin ½ particles having the intrinsic magnetic moments μ_p=+2.7927 and μ_n= -1.9131 nuclear magnetons. Their sum is thus $\mu_p+\mu_n=0.8796\ \mu_n$. This value differs only slightly from μ_d given above. The difference is $\mu_p+\mu_n-\mu_d=0.0222\ \mu_N$.

If we ignore this difference to a first approximation then we may expect the proton and neutron magnetic moment is aligned antiparallel in the deuteron. Since μ_n is negative, the neutron magnetic moment is aligned antiparallel to its intrinsic spin s_n. Hence the proton and neutron spins s_p and s_n must be aligned parallel to each other in the deuteron. The total intrinsic spin of the deuteron; $s_d=s_p+s_n=1$.

To the approximation in which the difference $\mu_{p+}\mu_n-\mu_d$ can be neglected, only the value L=0 is admissible. The other two values L=1 and 2 will contribute significantly to the magnetic moment due to the orbital rotation of the proton, which would introduce considerable difference between $\mu_p+\mu_n$ and μ_d.

Quadrupole moment of Deuteron

Another property which is highly important in connection with the shape of the nucleus is electric quadrupole moment. This quantity which cannot be discussed in a simple way, is a measure of deviation of a nucleus from spherical symmetry. Its magnitude depends on the size of the nucleus, the extend of deviation from spherical symmetry, and the magnitude of the charge ; the sign may be positive or negative.

When evaluating the quadrupole moment of deuteron only the proton contributes to the quadrupole moment and its distance from the centre of gravity is half of the proton neutron separation r. Hence the quadrupole moment operator for the deuterons is

$$Q=\frac{1}{4}\left(3Z^2-r^2\right)$$

$$=\frac{1}{4}\left(3r^2\cos^2\theta-r^2\right)$$

$$Q=\frac{r^2}{4}\left(3\cos^2\theta-1\right) \quad \text{...(1)}$$

The quadrupole is estimated from the 8 wavws beyond the potential well. The expectation value of this operator is given by

$$(\psi,Q\psi) = (\psi_s,Q\psi_s)+(\psi_D,Q\psi_D)+2(\psi_S,Q\psi_D) \quad \text{..............................(2)}$$

The S state is spherically symmetrical and cannot have a quadrupole moment. The first term is zero. The second term is apure D state term and is smaller than the cross term.

$$(\psi_D, Q\psi_D) = -\frac{1}{20}\int_0^\infty r^2\omega^2(r)dr$$

$$2(\psi_D, Q\psi_D) = \frac{\sqrt{2}}{10}\int_0^\infty r^2u(r)\omega(r)dr$$

Substituting in equation 2 we get,

$$Q = \frac{\sqrt{2}}{10}\int_0^\infty r^2u(r)\omega(r)dr - \frac{1}{20}\int_0^\infty r^2\omega^2(r)dr \quad ....(3)$$

The ground state of deuteron is predominating an s - state, the first term predominates over the second. Therefore the effective value of Q for the deuteron is given by

$$Q = \frac{\sqrt{2}}{10}\int_0^\infty r^2u(r)\omega(r)dr \quad ....(4)$$

U(r) is the deuteron ground state S wave function .

$u(r) = N_s e^{-\alpha r}$

$\omega(r)$ is the deuteron D wave function

$\omega(r) = N_s e^{-\alpha r}\left(1 + 3/\alpha r + 3/\alpha^2 r^2\right)$

N_S and N_D are normalization constants.

$$Q = \frac{\sqrt{2}}{10}\int_0^\infty r^2 N_D e^{-\alpha r} N_D e^{-\alpha r}\left[\left(1 + 3/\alpha r + 3/\alpha^2 r^2\right)\right]dr$$

Consider

$$P_s = \int_0^\infty u^2 dr = 1 \quad \text{and} \quad N_s = \sqrt{2\alpha}$$

$$Q = \frac{\sqrt{2}}{10}\int_0^\infty N_S N_D r^2 (e^{-\alpha r})^2 \left[\left(1 + 3/\alpha r + 3/\alpha^2 r^2\right)\right] dr$$

$$= N_S N_D / \sqrt{8}\alpha^3$$

$$Q = \frac{N_D}{2\alpha^{5/2}}$$

$$= N_S N_D / \sqrt{8}\alpha^3$$

$$N_D = 2Q\alpha^{5/2}$$

The function $\omega(r)$ outside the range of the force is determined completely by the quadrupole moment. The D state probability may be defined as

$$P_D = \int_0^\infty \omega^2(r) dr = 1$$

$$P_D = \int_0^{R_T} \omega^2(r) dr + \int_{R_T}^\infty \omega^2(r) dr$$

$$P_D = 2\int_{R_T}^\infty \omega^2(r) dr$$

Where RT is the nuclear range of the tensor potential

$$P_D = 2\int_{R_T}^0 9N^2{}_D \left(\frac{1}{\alpha_r}\right)^4 dr$$

$$P_D = \frac{16 N_D{}^2}{R_T{}^3 \alpha^4}$$

$$P_D = 24Q^2\alpha \Big/ R_T{}^3$$

This equation implies that the tensor force cannot have an arbitrarily small r

Neutron -Proton scattering at low energies

In principle the analysis of the scattering is very much altered by the non central force. At energies below 10 MeV it is advantageous to decompose the wave function into spherical harmonic by the use of spin angular function. The spin s can have values 0 and 1 corresponding to the singlet and triplet states respectively. The S scattering in the singlet state s=0 is unchanged by the presence of tensor forces since they do not act in that state.

The relative motion of two particles of masses M_1 and M_2 can be described by the wave equation.

$$-\frac{\hbar^2}{2\mu}\nabla^2\Psi + V(r)\Psi = E\Psi \qquad \ldots\ldots\ldots\ldots(1)$$

Where μ is the reduced mass. $E = E_L - E_C$ is the internal energy of the system. E_L is the L-system and E_C is the kinetic energy of the Centre of mass given

$$E_C = \frac{M_1}{M_1 + M_2} E_L \qquad \ldots\ldots\ldots\ldots (2)$$

For n-p scattering $M_1 = M_2 = M$ so that

$$E_C = E_L/2 \qquad \ldots\ldots\ldots\ldots (3)$$

So only half the laboratory energy is available for scattering in the Centre of mass system.

$$E = E_L - E_C = E_L/2 \qquad \ldots\ldots\ldots\ldots (4)$$

The angle of scattering θ_L in the L-system is related to that in the C system (θ_C) for n-p scattering

$\theta_C = 2\theta_L$

Also the angle between the neutron and the proton after scattering in the L-system is always 90^0

Since the reduced mass of the n-p system is μ =M/2, the wave equation (1) can be written as

$$\nabla^2\Psi + \frac{M}{\hbar^2}$$

Here $\Psi = \Psi(r, \theta, \Phi)$; θ and Φ are the centre of mass angles, r is the distance between the neutron and the proton. For scattering E>0.

The low energy n-p scattering cross section which can be used to calculate both σ_t and σ_s.

The conditions of matching the solutions of the radial wave equations for the deuteron ground state as also for low energy neutron-proton scattering are:

$$k^2 = \frac{M}{\hbar^2}E$$

$$k_{1}^{2} = \frac{M}{\hbar^2}\,0_t \qquad \text{and} \qquad k_{2}^{2} = \frac{M}{\hbar^2}\,0$$

Here we have written $E_{BE} = \varepsilon_t = \varepsilon_d$

Since both ε_t and E<<V_0, we can write

$$k_1^2 \approx k_2^2 \approx k_0^2 = \frac{M}{\hbar^2}v_0$$

Then from the above equations we have

$$-\alpha_t = k\cot(kb + \delta_0) \quad \text{.................................. (5)}$$

$$\cot(kb + \delta_0) = \frac{-\alpha_t}{k} \quad \text{.................................. (6)}$$

If we consider the scattering of neutrons of energy E = 1eV, then we get

$$\frac{\alpha_t^2}{k^2} = \frac{E_{BE}}{E}\;{}^{6}$$

$$\frac{\alpha_t}{k} = \ 1.5 \text{ x } 10^3 >>1 \qquad \text{………………………….…. (7)}$$

Thus cot (kb+δ_0) in Equation (6) has a large negative value which means that

$$kb + \delta_0 \approx \pi \qquad \text{…………………………… (8)}$$

For b = 2 fm, α_tb =0.464 and hence

$$kb = \frac{\alpha\alpha_t b}{1.5\times10^3}\frac{0.464}{1.5\times10^3} \qquad \text{……………………………………… (9)}$$

Comparing with Equation (8) we then conclude that kb<< δ_0 which gives

$$\cot(kb+\delta_0) \approx \cot\delta_0 = \frac{\alpha_t}{k} \qquad \text{……………………………………. (10)}$$

We then get the low energy triplet scattering cross section as

$$\sigma_t = \frac{4\pi}{k^2 + k^2\cot^2\delta_0}\frac{4\pi}{k^2 + \alpha_{t^2}}\frac{4\pi\hbar^2}{M}\frac{1}{E+\varepsilon_d} \qquad \text{…………………… (11)}$$

An expression similar to Equation (11) can be written for the singlet scattering cross section σ_s for low energy neutrons:

$$\sigma_s = \frac{4\pi}{k^2 + \alpha_s^2} = \frac{4\pi\hbar^2}{M}\frac{1}{E+\varepsilon_s} \quad . \qquad \text{………………………………… (12)}$$

Here σ_s^2= (M/$\hbar^2$) ε_s, ε_s being the energy of the singlet state of the neutron – proton system, analogous to the triplet state energy $\varepsilon_d = E_{BE}$

We can then write the low energy n-p scattering cross section as

$$\sigma = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_s = \frac{\pi\hbar^2}{M}\left(\frac{3}{E+\varepsilon_d} + \frac{1}{E+\varepsilon_s}\right) \qquad \text{………………… .(13)}$$

Equation (13) gives the zero energy scattering cross section as

$$\sigma_0 = \frac{\pi\hbar^2}{M}\left[\frac{3}{\varepsilon_d} + \frac{1}{\varepsilon_s}\right] \qquad \text{………………………..……… (14)}$$

We can then estimate the energy ε_s of the singlet state, using the measured value of the zero energy n-p cross section. Substituting ε_d = 2.226 M_eV and σ_0 =20 b in the above equation, we get ε_s = 90 KeV

Proton - Proton Scattering at Low Energy

The quantitative information about the spin dependence is confirmed by analyzing the scattering of low energy protons from protons. This analysis also shows that the nucleon potential is charge independent , ie. the same nuclear potential may be used for both np and pp scattering This doesn't mean that their cross section are the same. There are two differences between pp and np scattering . First pp scattering is caused not only by nuclear force but also by Coulomb's force. Second the scattering and scattered particles are identical and obey Pauli's exclusion principle and therefore wave function S describing the two protons must change sign on the interchange of two particles. For incident protons below 10 MeV only S state interaction is of any importance in the scattering. Since proton in higher orbital angular momentum states stay apart from each other beyond the range of nuclear force.

Experimentally the pp scattering , is capable of much higher accuracy than in np scattering for the following reasons.

1. Protons are easily available over wide range of frequency

2. Protons can be made mono energetic.

3. Protons can be produced in well collimated beam.

4. Protons can be easily detected by their ionizing properties.

5. Protons can undergo both Coulomb and nuclear scattering

6. The protons combination obeys Fermi Statistics.

Hence this simplifies the analysis of proton- proton scattering

We are now interested in obtaining theoretical expression for differential elastic scattering cross section of protons by protons. The theory is more complicated than np scattering because Coulomb's potential distort the incident wave even at finite distances. The particles incident along the z axis the total wave function is given by

$$\psi(r,\theta)=\sum C_l \frac{u_l(r)}{r} P_l(\cos\theta)$$

Where $u_l(r)$ is the solution of the radial wave equation for the proton –proton system given as

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}\left[E-V(r)-\frac{e^2}{4\pi\varepsilon_0 r}-\frac{l(l+1)\hbar^2}{Mr^2}\right]u=0$$

.Where V(r) is the square well nuclear potential and M is the mass of the proton. For low energies the nuclear potential will affect only to l =0 partial wave, Coulomb potential produces higher l value scattering .

Meson theory of nuclear force

Nucleons are held together by the exchange of intermediate nuclear particle known as mesons.

According to Yukawa theory the particles are intermediate in mass between electrons and nucleons are responsible for the nuclear forces. These particles are known called as pios. Pions may be charged π^+, π^- π^0 and are members of the class of elementary particles called as mesons. The word pion is a contraction of original name pi meson.

According to Yukawa theory every nucleon continually emits and reabsorbs pions. Once emitted pion is absorbed by another nucleon not by the parent nucleon. The associated transfer of momentum is equivalent to the action of nuclear force. Nuclear forces are repulsive at very short range as well as being attractive at a greater nuclear distances(~ 2 Fermi). One of the strength of meson theory of such nuclear forces is that it can account for both these properties.

This is like two boys exchanging basket balls if they throw the ball at each other, boys move backward ,and when they catch the ball thrown at them, the backward momentum increases. This method of exchanging balls has the same effect as the repulsive force between the boys. If the boys snatch the ball from each other's hand ,however , the result will be equivalent to an attractive force between them . If nucleons constantly emit and absorb pions their mases expected to decrease, but it does not happen.

This is justified by uncertainty principle. The laws of physics refer to measurable quantities only and the uncertainty principle limits the accuracy with certain combination of the measurement can made. The emissions of pion by the nucleon which does not change in mass., this is the clear violation of law of conservation of energy. So nucleons reabsorbs it or absorbs another pions emitting by the neighboring nucleon so soon after ward that even in principle it is impossible to determine the mass change involved . From uncertainty principle in the form $\Delta E\ \Delta t \geq \hbar/2$. An event in which an amount of energy ΔE is not conserved is not prohibited so long as the duration of event does not exceed $\frac{\hbar}{2.E}$

For this condition let us estimate the pion mass. Let us assume the pions travel between the nucleus at the speed of v very close to the velocity of light c, (v <c).

The emission of pion mass m_π represents the temporary energy discrepancy of

$\Delta E \sim m_\pi c^2$ (this neglects the pion kinetic energy). And that $\Delta E \Delta t \sim \hbar$.

Nuclear forces have maximum range of about 1.7 fermi and the time Δt needed for the pion to travel is $\Delta t = \frac{r}{v} \approx \frac{r}{c}$

Therefore we have $\Delta E \Delta t \sim \hbar$

$$m_\pi c^2 x c \frac{r}{c} \approx \hbar$$

$$m_\pi \approx \frac{\hbar}{rc}$$

This gives the value of $m_\pi \sim 2 \times 10^{-28}$ kg

This is roughly 220 times the rest mass of electron.

Reciprocity Theorem:

Let as consider a reversible process X + x = Y + y, in which X, x, Y and y occur in arbitrary numbers in a large box of volume V.

We are interesting in the relation between the total cross-section σ(x → y), most generally σ (α → β) of the reaction with entrance channel α and reaction channel β and the total cross-section σ (β → α) of the inverse reaction. For this we use the fundamental theorem of statistical mechanics (*the principle of overall balance*), which states that when the system is in dynamical equilibrium all energetically permissible states are occupied with equal probability. Here we are interested in two particular states, the reaction channels α and β . The theorem is then equivalent to starting the given energy range the number of possible channels in the box is proportional to the number of possible channels into the box. The latter is given by

$$N_\alpha = \frac{4\pi P_\alpha^{\ 2} V dp_\alpha}{h^3} = \frac{P_\alpha^{\ 2} V dp_\alpha}{2\pi^2 h^3} \quad \text{...(1)}$$

Since v = dE/dp, hence $N_\alpha = \dfrac{P_\alpha^{\ 2} V dE_\alpha}{2\pi^2 h^3 v_\alpha}$.

Similarly, we have $N_\beta = \dfrac{P_\beta^{\ 2} V dE_\beta}{2\pi^2 h^3 v_\beta}$.

The energy range of the two channels must be the same, ie. $dE_\alpha = dE_\beta$

that is number of channels α in the box/ number of channels β in the box

$$\frac{N_\alpha}{N_\beta} = \frac{P_\alpha^{\ 2} V_\beta}{P_\beta^{\ 2} V_\alpha} \quad \text{...(2)}$$

The system is in dynamical equilibrium when the number of α→β transitions per second is equal to the number of β → α transitions per second. The condition usually holds and is known as the *Principle of detailed balance.* Further

No. of transitions α→β per sec = $N_\alpha \times T(\alpha \to \beta)$, Where $T(\alpha \rightarrow \beta)$ is the transition probability for the transition $\alpha \rightarrow \beta$

Hence $P_\alpha^2 V_\beta T(\alpha \rightarrow \beta) = P_\beta^2 V_\alpha T(\beta \rightarrow \alpha)$ ………………..(3)

The transition probability measures the chance that one particle moving with velocity *v* in volume V is scattered per sec. Hence the cross-section σ which corresponds to unit incident flux is given by the relation

σ = TV/*v*. ………………………………………..(4)

Combining relations (3) and (4) and using k = p/ ħ , we have

$$k_{\alpha^2} \sigma(\alpha \rightarrow \beta) = k_{\beta^2} \sigma$$

$$\sigma(\alpha\rightarrow\beta)\lambda^2_\alpha = \sigma(\beta\rightarrow\alpha)/\lambda^2_\beta$$

We have assumed zero intrinsic angular moments for the particles so far. If J is the intrinsic angular momentum of any one of the particles, the corresponding density of states then must be multiplied by 2J + 1. Thus if there are intrinsic momenta for X, x, Y and y, we may write

$$(2J_X + 1)\,(2J_x + 1)\,k^2_\alpha\sigma(\alpha\rightarrow\beta) = (2J_Y + 1)\,(2J_y+1)\,k^2_\beta\,\sigma\,(\beta\rightarrow\alpha).$$

If the initial and final states have definite angular momenta, then the above equation must be employed.

Breit - Wigner Single level formula for scattering

To study the nuclear reactions, it is necessary to have the quantitative measure of probability of a given nuclear reaction. This quantity must be one which can be measured experimentally and calculated in such a way the theoretical and experimental values can be compare readily. The quantity that is most often used for this purpose is the cross section of the nucleus for a particular reaction usually denoted by σ with appropriate subscript. The concept of nuclear cross section can be easily visualized as the cross sectional area or target area, presented by the nucleus to an incident particle.

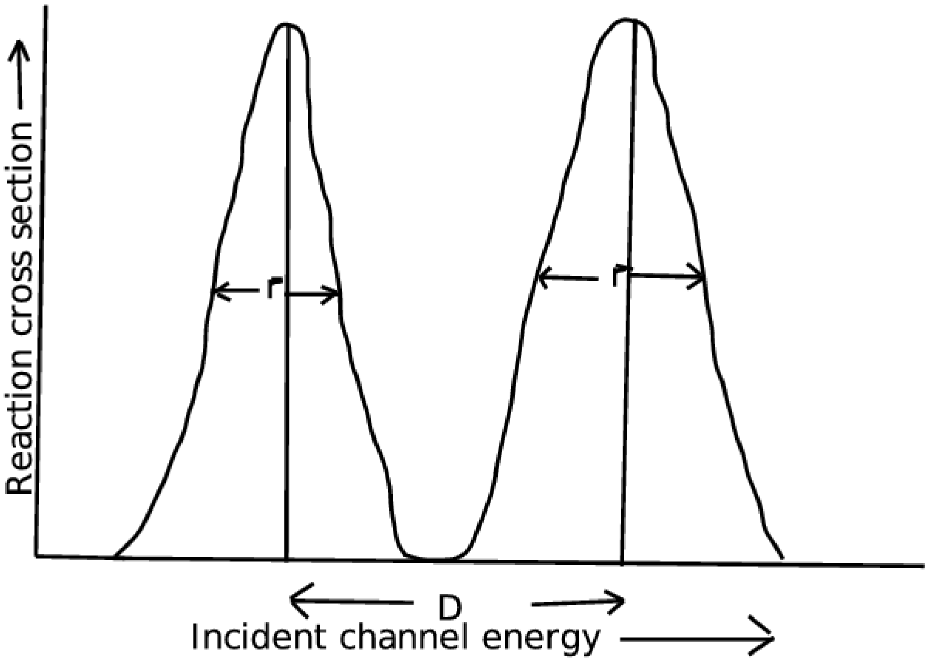


Figure: Reaction cross section as a function of incident channel energy

A nuclear reaction produced by the absorption of the projectile x by a target nucleus X (both in their ground state) to produce a compound nucleus C* in a state of excitation near an isolated level of the latter which is far removed from any of its other levels. The existence of such an isolated level implies that separation between the levels D >> Γ; Γ being width of the level. The reaction is initiated through a definite entrance channel (X + x) which is characterized by a definite value of the kinetic energy of relative motion E_x between X and x and of a definite relative angular momentum.

The energy of excitation of the compound nucleus, so formed is given by

$E_c = E_x + S_x = Er$ .... (1)

Where, E_r is the energy of the isolated level in which the compound nucleus is formed. S_x is the separation energy of x from the compound nucleus in its ground state, given by

$S_x = B_c - B_X - B_x$ ...... (2)

Where the B's denote the binding energies of the corresponding nuclei. If x is a nucleon, then $B_x = 0$. The subsequent breakup of the compound nucleus into Y+y with the relative kinetic energy E_y .Obviously we can also write $E_c = E_y + S_y$ where S_y is the separation energy of y from the compound nucleus in the ground state, given by.

$S_y \quad = \quad B_c - B_Y - B_y$

If y is nucleon, $B_y = 0$. Here we assume that both Y and y are formed in their ground states. However, this may not always be the case and Y may be left in different excited states, giving rise to the different possible exit channels.

The state of the compound nucleus formed as above can be represented by a damped harmonic wave given below.

$$\Psi(t) = \psi_o \exp\left(-iE_r\ t/\hbar\right) \exp\left(-\Gamma t/2\hbar\right)$$

$$= \Psi_o \exp\left\{-i\left(E_r - \frac{i\Gamma}{2}\right)t/\hbar\right\} \qquad \text{.............. (3)}$$

Here Γ /2 is the half width of the level which is actually a decaying state, its life - time being $\tau = \hbar/\Gamma$

The above wave function does not represent a stationary state, but may be through of a being built up by the super position of stationary states of different energies by Fourier integral method.

$$\psi(t) = \int_{-\infty}^{+\infty} A_E \exp\left(-iEt/\hbar\right) dE \qquad \text{................... (4)}$$

We can determine the amplitude A_E of the state at the energy E by taking the Fourier transform of equation (4).

$$A_E = \frac{1}{2\pi}\int_0^{\infty} \psi\left(t^1\right) \exp\left(iEt'/\hbar\right) dt'$$

$$= \frac{1}{2\pi}\int_0^{\infty} \Psi_o \exp\left\{i(E - E_r + i\Gamma/2)t'/\hbar\right\} dt'$$

$$= \frac{\psi_o}{2\pi}\left[\frac{\exp\left\{i\left(E - E_r + i\Gamma/2\right)t'/\hbar\right\}}{i(E - E_r + i\Gamma/2)/\hbar}\right]_0^\infty$$

Here we take positive value of time only, since the compound nucleus can decay only after its formation.

$$\therefore A_E = \frac{\psi_o}{2\pi}\frac{i\hbar}{\left(E - Er + i\Gamma/2\right)}$$

The upper limit of the above integral vanishes because of the damping term $\exp\left(-\Gamma t'/2\hbar\right)$. So we get

$$\left|A_E\right|^2 = \frac{\left|\psi_o\right|^2}{4\pi^2}\frac{h^2}{\left(E - E_r\right)^2 + \Gamma^2/4}$$

The cross section for the formation of the state E_c is by the process X + x proportional to the amplitude squared. So we can write.

$$\sigma_x = \frac{C}{\left(E - E_r\right)^2 + \Gamma^2/4}$$

Where C is a constant .To determine C we note that the number of possible states in the incident channel is , $dn = \frac{4\pi p_x^2 \Omega dp_x}{\left(2\pi\hbar\right)^3}$

Where Ω is the volume of the enclosure within which the reaction take place. If σ_x is the cross section for the absorption of x by X, then the volume swept out by the effective collision area in one second is σ_x v_x where v_x is the relative velocity of the incident particle . So the probability of finding the nucleus X in this volume is σ_x v_x/Ω and the probability of formation of the compound nucleus in the given entrance channel per second is

$$\frac{\sigma_x v_x}{\Omega}\cdot\frac{4\pi p_x^2 \Omega dp_x}{\left(2\pi\hbar\right)^3} = \frac{\sigma_x p_x^2 v_x dp_x}{2\pi^2\hbar^3} = \frac{\sigma_x p_x^2 v_x dE_x}{2\pi^2\hbar^3}$$

Integrating over all possible energies we get the total probability as

$$P = \frac{1}{2\pi^2\hbar}\int_{-\infty}^{+\infty} \frac{\sigma_x}{\lambda^2}\, dE_x$$

Since the integrand has finite values only for the energies within the width Γ of the level which are so narrow that we can neglect the variation λ and write (dE = dE_x)

$$P = \frac{1}{2\pi^2\hbar\lambda^2}\int_{-\infty}^{+\infty} \sigma_x\, dE$$

$$= \frac{C}{2\pi^2\hbar\lambda^2}\int_{-\infty}^{+\infty} \frac{dE}{(E - E_t)^2 + \Gamma^2/4}$$

$$= \frac{C}{2\pi^2\hbar\lambda^2} \cdot \frac{2\pi}{\Gamma} = \frac{C}{\pi\hbar\Gamma\lambda^2}$$

The above probability of the formation of the compound nucleus must be equal to the probability of the decay of C* through the same channel. This is a consequence of the reciprocity theorem. If we write this probability of decay through the entrance channel as $\Gamma_x/\hbar$ we get.

$$\frac{C}{\pi\hbar\Gamma\lambda^2} \cdot \frac{\Gamma_x}{\hbar} \qquad \text{Or} \qquad C = \pi\lambda^2\Gamma_x\Gamma$$

So we finally get the cross section for the formation of the compound nucleus as

$$\sigma_x = \frac{\pi\lambda^2\Gamma_x\Gamma}{(E - E_t)^2 + \Gamma^2/4}$$

The relative probability of decay of C* through the exit channel Y+y can be written as Γ_y/Γ. We then get the cross section for the reaction X(x,y) Y as

$$\sigma(x,y) = \sigma_x \frac{\Gamma_y}{\Gamma} = \pi\lambda^2 \frac{\Gamma_x\Gamma_y}{(E - E_r)^2 + \Gamma^2/4}$$

This is the Breit Wigner one level formula for spinless nuclei at very low energies so that the relative angular momentum of the particles in the entrance channel is l =0. If however l is not zero, which is the case when the energy is higher, we have to take into account the statistical factor of the compound state formed which, for spinless nuclei x and X, is given by g = 2 l +1

Each of the (2 l +1) sub states can decay with equal probability. So Γ_x is to be multiplied by this factor, which gives.

$$\sigma_x^l = \pi \bar{\lambda}^2 \left(2l+1\right) \frac{\Gamma_x \Gamma}{\left(E-E_r\right)^2 + \Gamma^2/4}$$

$$\sigma^{(l)}\,(x,y) = \pi \bar{\lambda}^2 \left(2l+1\right) \frac{\Gamma_x \Gamma_y}{\left(E-E_r\right)^2 + \Gamma^{2\cdot}/4}$$

UNIT IV : NEUTRONS

Neutron source - properties of neutron - , charge , spin and statistics, decay, magnetic moment – classification of neutrons – neutron diffusion – neutron current density, neutron leakage rate, thermal neutron diffusion, fast neutron diffusion ,Fermi age equation –nuclear reactors, nuclear chain reaction - four factor formula – general aspects of nuclear reactors – classification of nuclear reactors

Neutron Sources

1. Radioactive Sources

The source consists of a mixture of about 5 parts of fine beryllium powder to 1 part of radium. The majority of the neutrons is produced by the (α , n) reaction given above. If the radium is in secular equilibrium with its decay products, extra deuteron are released according to the reaction.

$_4Be^9 + _2He^4 \rightarrow _6C^{13*} \rightarrow _6C^{12} + _0n^1$.

The product nucleus of this reaction $_6C^{13*}$, is unstable and thus decays into $_6C^{12}$ and $_0n^1$ within 10^{-15} sec. This neutron source is isotropic and produces neutrons over a wide energy range (0 - 13 MeV). The neutrons are not mono - energetic due to following reasons:

a) The α-particles do not have the same initial energy.

b) Many lose part of their initial energy by ionization before nuclear capture.

c) The neutron energy varies with the direction of emission having least value if emitted in the direction opposite to that of the incident α -particles.

(d) $_6C^{12}$ can be left in an excited state.

We can have slow neutrons after putting the Ra-α-Be source in water, paraffin, or some other hydrogen containing material. For a 2-MeV neutron only 25 elastic collisions with protons are needed. Radon is sometimes used instead of radium, as it is a gas and can be made into a more compact source. But it has the disadvantage of decaying rapidly because of the short half life of 3.8 days.

2. Photo-neutron Sources

When gamma rays of sufficient energy (greater than the neutron binding energy) are incident upon light elements a probability exists that neutrons will be emitted by the energized nuclei. The majority of nuclei have a neutron binding energy of 5 MeV or more, except $_4Be^9$ (1.67 MeV) and $_1H^2$(2.23 MeV), gamma -rays from artificially produced radio - nuclei can serve in photo disintegration sources. Yield, the number of neutrons per sec due to 1 gm. of Be or D_2O at a distance of 1 cm. from 1 curie, of various gamma sources is given in the table

Table : Radioactive (gamma, n) sources)

Gamma(γ) Source	Target	Half Life	E_γ (MeV)	E_n (MeV)	Yield
Na^{24}	Be	14.8 hr.	2.76	0.83	$13x10^4$
Na^{24}	D_2O	14.8 hr.	2.76	0.22	$27x10^4$
Y^{88}	Be	87 d.	1.9, 2.8	0.16	$10x10^4$
Y^{88}	D_2O	87 d.	2.8	0.31	$0.3x10^4$
Sb^{124}	Be	60 d.	1.7	0.025	$19x10^4$

The advantages of this type of source are : a) The reproducible neutron strength. b) The choice of E_o by choosing a suitable value of E_γ . c) The wide variety of artificial γ -sources available from atomic reactors. d) The neutrons produced are more mono -energetic those obtained from (Be- γ) sources.

3. Neutrons from Accelerated Particle Reactions.

The invention of the Van de Graaff machine, cyclotron and other charged particle accelerators led to the rapid development of new neutron sources. Neutrons are produced by (d,n) and (p,n) reactions in particle accelerators. These sources give mono - energetic neutrons in the range from a few keV to 20 MeV. In deuteron bombardment the energy of the released neutron is progressively larger for most of the target elements, which is due to the low binding energy of the deuteron.

The C^{12} (d, n) N^{13} reaction is an exception to the rule with a Q value of - 0.28 MeV. Those neutrons which are emitted in the direction of the incident beam of deuterons will have a maximum energy, equal to the sum of the incident and the reaction energies approximately. Some (d, n) reactions are:

$${}_1H^2 + d \longrightarrow {}_2He^3 + {}_0n^1 + 3.265 \text{ MeV}$$

$${}_1H^3 + d \longrightarrow {}_2He^4 + {}_0n^1 + 17.6 \text{ MeV}$$

$${}_3Li^7 + d \longrightarrow {}_4Be^8 + {}_0n^1 + 15.0 \text{ MeV}$$

Several light and moderately heavy elements undergo (p,n) reactions. These endoergic reactions are the convenient sources of neutrons of moderate energy. Some (p, n) reactions are:

$${}_1H^3 + p \longrightarrow {}_2He^4 + {}_0n^1 + 0.764 \text{ MeV}$$

$${}_3Li^7 + p \longrightarrow {}_4Be^7 + {}_0n^1 + 1.646 \text{ MeV}$$

The Li^7 (p,n) Be^7 reaction suggests that accelerated charged particles can be used to produce nearly mono energetic neutrons.

4. Ultra fast Neutrons

As a deuteron behaves as a relatively loosely bound system consisting of a neutron and proton which are frequently outside the range of their mutual forces, hence, deuterons accelerated to about 200 MeV in a synchro-cyclotron can readily be "stripped" into protons and neutrons by directing the deuteron beam to pass through a target nucleus. Recently neutrons of energy > 300 MeV have been produced by stripping 650 MeV deuterons accelerated in the synchrotron. Almost any element may be used as the target material for stripping but the yield varies with its nature.

5. X - rays Sources

Through the (γ, n) reaction, high energy X-ray machines can produce the high energy neutrons. A flux of high energy photons is produced when the electrons accelerated to high energies in linear accelerators, betatrons, synchrotrons etc. impinge on target elements of high atomic number. A large number of neutrons within the target material are released by the high flux of photons. The disadvantage of this type of source is the wide spread in neutron energy.

6. Reactor Sources

The most powerful source of neutrons available in these days is a nuclear reactor or chain reacting pile on which uranium undergoes fission. The fission of each heavy nucleus by slow neutron capture is accompanied by the release on the average of about 2.5 neutrons. The neutrons produced from the fission of uranium atoms are fast and are slowed down in the moderator. The disadvantages of reactor sources are the wide spread in neutron energy and the impossibility of directly pulsing the neutron source.

Basic Properties of the Neutron

1. Neutron mass

Since the neutron is not a charged particle, its mass could not be determined from deflection measurements in electric and magnetic fields. Chadwick gave a rough estimation of the mass of a neutron from the information obtained in collisions of neutrons with protons on one hand and with nitrogen nuclei on the other. From the equations for the conservation of energy and for the conservation of momentum for a head on collision, and from the lengths of accompanying tracks in the cloud chamber, Chadwick found that neutron mass $m_n = 1.15$ amu, roughly the same as that of the proton.

A more precise value for the mass of the neutron was obtained by Chadwick from the study of the nuclear reaction.

$${}_5B^{11} + {}_2He^4 \dashrightarrow {}_7N^{14} + {}_0n^1 + Q.$$

The Q-value was found from the kinetic energies of B^{11}, N^{14}, $_0n^1$ and α -particles, Substituting all the known values in terms of atomic mass units in the above equation Chadwick obtained 1.0067 amu. for the mass of the neutron.

2. Neutron Charge

The neutron is usually assumed to have no net charge. Dee investigated the ionization produced in air in a cloud chamber irradiated by fast neutrons and found one ion pair per 3 metre of the path and thus concluded that the charge of the one neutron is less than 1/ 700 of the proton charge. Rabi and his coworkers have found the neutron charges less than 10^{-12} electron charge. Fermi and Marshall set an upper limit of 10^{-18} electron charge for the charge on the neutron.

3. Neutron Statistics and Spin

Since the deuteron is a loose combination of proton and neutron and is supposed to have a spin 1. From the study of band spectra of deuterium and the proton spin (½) the neutron spin comes out to ½. Neutrons have been found to obey Fermi statistics. This value of ½ for the spin of the neutron has been confirmed by reflection of neutrons from magnetised mirrors by Hughes and Burgy.

4. Decay of the neutron

The first accurate determination of the mass of neutron by Chadwick showed that the neutron mass was greater than that of a proton. It was also evident that the neutron outside the nucleus is unstable and could be expected to decay with the emission of a beta particle plus an anti neutrino ($\bar{\nu}$), leaving a proton.

n---> p + e^- + $\bar{\nu}$ + 782 keV.

5. Half Life of Neutron

The estimate of the half life of the neutron was obtained from a determination of the density of the neutron beam and the number of neutrons decaying per unit time per unit volume. The neutron density was obtained by exposing manganese foils to the neutron beam and measuring their activities against the activities for the calibrated standard foils of identical thickness. At the centre of the beam the value so obtained was 1.6 x 10^{10}

neutrons/m^3. The number of neutrons decaying per unit time per unit volume was estimated from the number of protons per unit time striking the first electrode of the electron multiplier and the volume of the beam where the protons originate. The value so obtained was 6.3 x 10^8 neutrons/ min /m^3 of the neutrons beam. Therefore, the half life $T_{1/2}$ becomes.

$T_{1/2}$ = 1.16x 10^{10} x 0.693 / 6.3 x 10^8 = 12.8 min.

6. Wave properties

For non - relativistic velocities (v<<c), the wavelength associated with neutrons is given by

$$\lambda = \frac{h}{p} = \frac{h}{(2ME)^{1/2}} = \frac{2.86x10^{-11}}{E^{1/2}} metre$$

Where E is in eV. Thus for thermal neutrons λ= 1.82 A^o which is comparable with atomic dimensions. For E=1MeV, λ = 2.86 x 10^{-14} metre approaching nuclear dimensions. The relativistic correction is important above energy of 100 MeV. The wave properties are of primary importance in determining the nature of the interaction between neutrons and nuclei.

7. Magnetic moment of the Neutron

It is sometimes said that the neutron should not have a magnetic moment because it has zero charge. Actually magnetic moments depend upon currents rather than charges. For example the hydrogen atom has no net charge but does have a magnetic moment. Thus neutron can be supposed to have a magnetic moment if it contains equal +ve and -ve charges which are in motion. According to Yukawa's meson field theory neutron transforms into a proton and a negative π- meson for a fraction of time. Since this meson is lighter than the proton, its motion will be faster and the resultant magnetic moment is that of revolving a negative charge. Hence it is negative. The proton and π meson also possess spin and, therefore, have intrinsic magnetic moments, that of the meson being large due its small mass. This situation also contributes to a negative moment of the neutron.

The -ve sign indicates that the magnetic moment of the neutron is similar to that generated by rotating - ve charge with its spin parallel to that of the neutron.

Since the deuteron is believed to consist of a proton and a neutron with parallel spins, the magnetic moment should be equal to the sum of the separate nucleon moments. But this sum differs from deuteron moment by 0.022 nuclear magneton. This discrepancy may be due to following reasons.

a. The deuteron spends part of its time in a state of higher orbital momentum, in which its magnetic moment would be definitely less than the ground state.

b. There would be a difference in the magnetic moment of the nucleons in the free and combined conditions.

Classification of Neutrons

The neutron energy can be determined by measuring the ranges projected by them in an ionization chamber. For more detailed examination of the results of the interactions of neutrons with matter it is also desirable to classify them according to their kinetic energy.

1. Slow Neutrons

Neutrons with energies from zero to about 1000 eV are usually included in this category. In this range sub- classifications occur. The most important are given below.

a. Cold Neutrons

Neutrons having their average energy less than thermal neutrons are called cold neutrons. These neutrons are produced by a device depending on the coherent scattering. From the Bragg law ($n\lambda = 2d \sin\theta$) we have no reflection if the neutron wavelength λ exceeds 2d, where d is the grating spacing of the crystal. Thus the neutrons with the lower energies are transmitted. Since 2d for graphite is 6.7 A^o ,the computed maximum energy of the transmitted neutrons is 0.002 eV, which is well below the average energy of thermal neutrons.

b. Thermal Neutrons

When fast neutrons have been slowed down until the average energy of the neutrons is equal to the average thermal energy of the atoms around them, the neutrons are called thermal neutrons. At each collision with these atoms, the neutron may gain or loss energy. The maximum number of neutrons will have the energy kT, which is about 0.025 eV at 20^{o}C.

c. Epithermal Neutrons

Consider an arrangement in which a fast neutron source is placed inside a moderator which slows the neutrons down until they are in equilibrium with the molecules of moderator. Before establishing the thermal equilibrium, the distribution of velocities will contain velocities which exceed any permitted by a Maxwell distribution for the temperature of the moderator. Such a distribution is called epithermal and the neutrons in it are called epithermal neutrons.

d. Resonance Neutrons

Since the various nuclei exhibit strong absorption of neutrons at fairly well defined energies in the range of energies between 1 to 100 eV. These absorptions are known as resonance absorptions and the neutrons having the corresponding energies are known as resonance neutrons.

2. Intermediate Neutrons

Intermediate neutrons in the energy region between 1000 eV and 0.5 MeV are obtained by the deceleration of fast neutrons. We have less information about intermediate neutrons than about slow neutrons because of the difficulty of finding efficient detectors. In this energy range elastic scattering process is dominant. Recently a number of techniques have been development for the study of this energy range.

3. Fast Neutrons

Neutrons with energies having range between 0.5 to 10 MeV are called fast neutrons. This energy region is characterized by the appearance of many nuclear reactions which are energetically impossible at lower neutron energies of which the most important is inelastic scattering.

4. Very Fast Neutrons

This energy interval (10-50 MeV) is distinguished from the preceding by the appearance of nuclear reactions involving the emission of more than one product, such as the (n, 2n) reaction.

5. Ultra Fast Neutrons

Neutrons with energies beyond 50 MeV are called ultra high energy neutrons. They are produced in the target by p-n interactions induced in nuclei by high energy protons. The cosmic radiation is also a source of neutrons with energies well above those which are likely to be produced by accelerations.

Neutron Diffusion

Diffusion is the process that takes place when particles of one type move between particles of another type into which they are penetrating. The energy loss suffered by the neutrons as a consequence of collisions with the nuclei of the moderator. It is often necessary to know the spatial distribution of the neutrons, i.e., the dependence of the neutron density on position. The problem can be treated with the aid of the theory of diffusion. This is not new in physics but is similar to the problem of the diffusion of a gas through another gas or to that of the diffusion of electrons though a gas. For the sake of simplicity we make several assumptions. i) The neutron flow does not change with time (ii) we assume that neutron absorption is negligibly small. (iii) In laboratory system, the probability of a particular scattering angle is a constant. (iv) The neutrons are monoenergetic and the scattering is entirely due to elastic collisions. The result of calculations based on these assumptions is in fairly good agreement with experiment.

Neutrons current density

If the neutron density is not constant throughout a given volume, neutrons move from regions of higher density to regions of lower density. If we consider a unit area in a volume occupied by a neutron gas. There will be an excess of neutrons passing through this area in one direction as compared to the number of neutrons crossing it in the opposite direction. When the neutron density becomes uniform in the neighborhood of our unit area, the net flow stops. The net number of neutrons travelling per second through the unit area which is normal to the direction of flow is known as neutron current density. It is a vector quantity and can be started in Cartesian coordinates as:

$$J = J_x i + J_y j + J_z k.$$

According to Fick, neutron current Density J and the space rate of change of the neutron density dn/ds are proportional to each other, $J = -D\, dn/ds$.

Here D is the diffusion coefficient and negative sign indicates that the neutron current is in the direction of decreasing neutron density.

Neutron Leakage Rate

Let us consider a small volume element dV = dxdydz in a rectangular frame of reference. The neutron leakage rate can now be calculated as the difference between the number of particles entering and the number of particles exiting. Let us first consider a direction parallel to the x-axis. The number of particles crossing the face dy dz is J_x dy dz. After entering they continue to move in the same direction and at a distance dx they encounter the opposite face of volume element. If $\partial J_x/\partial x$ is the rate of change of the current density per unit length along the x-axis, the total number of particles crossing the opposite face of volume element dV will thus be

$$[J_x + (\partial J_x/\partial x)\, dx]\, dy\, dz.$$

Hence the net leakage rate of neutrons from the volume element in x-direction

$$= (\partial J_x/\partial x)\, dx\, dy\, dz.$$

The same calculations lead to the leakage from the volume element in y- and z-directions. Thus the total leakage rate $= (\partial J_x/\partial x + \partial J_y/\partial y + \partial J_z/\partial z)\, dx\, dy\, dz$

$$= - D_o\, (\partial^2\Phi/ \partial x^2 + \partial^2\Phi/ \partial y^2 + \partial^2\Phi/ \partial z^2)\, dx\, dy\, dz$$

$$= -D_0 \nabla^2 \phi dV = -D \nabla^2 n dV$$

The ∇^2 is used for the sum of second derivatives of a function and is known as the Laplacian. In spherical and cylindrical coordinates respectively it is given by

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\left[\frac{\partial^2}{\partial\varphi^2} + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial}{\partial z^2}$$

Thermal Neutron Diffusion

To simplify the problem, let us treat thermal neutrons as a mono energetic group. All the neutrons of the group are assumed to have same energy. On the average net energy change remains zero when colliding with the nuclei of moderator. At a given point r (x, y, z) of the moderator, the neutron density n(r) will depend upon following three factors: (a) The rate of production of thermal neutrons per unit volume, Q.

(b) The rate of absorption of thermal neutrons per unit volume, $nv\sum a$

(c) The rate of diffusion or leakage per unit volume,. $-D\nabla^2 n$

Hence the rate of change of neutron concentration with time ∂n/∂t = rate of production - rate of absorption - rate of leakage

$$= Q - nv\sum a\left(-D\nabla^2 n\right)_{a_{tr}^2} \quad \text{------- (1)}$$

This is the diffusion equation for neutrons that has to be solved for each particular case according to the requirement of the problem. When a steady state has been established the neutron density n at any given point inside the moderator will be independent of time t. This is in fact reached after enough time has elapsed, so that the production rate of neutron and the leakage and absorption have adjusted themselves to an equilibrium value. Since production of thermal neutrons is mainly due to slowing down of fast neutrons to thermal energies, hence Q can be replaced by slowing down density q. Thus steady state equation can be written as

$$\nabla^2 n - -\frac{3}{\lambda_{tr}\lambda_a} n + \frac{3q}{v\lambda_{tr}} = 0 \quad \text{--------------- (2)}$$

In terms of neutron flux ϕ we can write

$$\nabla^2\phi - \frac{3}{\lambda_{tr}\lambda_a}\varphi + \frac{3q}{\lambda_{tr}} = 0 \quad \text{---------------- (3)}$$

Let us imagine that the source is a point. We can imagine surrounding the source point a sphere of infinitesimal radius so that in all the space outside it, the q = 0. Thus the diffusion equation for these regions reduces to

$$\nabla^2\phi - \frac{3}{\lambda_{tr}\lambda_a}\varphi = 0 \quad \text{or} \quad \nabla^2\phi - \frac{1}{L^2}\varphi = 0 \quad \text{------------(4)}$$

Where $L^2 = \lambda_{tr}\lambda_a /3$. This term L is known as thermal diffusion length. To get a solution of eqn (4), the ∇^2 is expressed in coordinates most appropriate to the geometry of the problem.

We shall write the diffusion equation for the spherical reference system, making use of the Laplacian in spherical form for the region outside the point source. The equation is hence.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{L^2}\varphi = 0$$

After substituting $R = r\phi$, we get

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dR}{dr} - R\right) - \frac{1R}{L^2 r} = 0, or \frac{d^2R}{dr^2} - \frac{R}{L^2} = 0$$

This equation has the solution

$R = Ae^{r/L} + Be^{-r/L}$ OR

$$\varphi = \frac{A}{r}e^{r/L} + \frac{B}{r}e^{-r/L}$$

These arbitrary constants A and B, can be determined from the boundary conditions. Since the neutron flux is finite for all values of the variable r, hence the constant A must be zero. To determine B we can make use of the physical condition that the total neutron absorption rate for the medium must be equal to the source strength Q. Therefore

$$Q = \int_0^\infty \phi\Sigma_a \, dV = \int_0^\infty \phi\Sigma_a \, 4\pi r^2 \, dr$$

$$= \int_0^\infty \frac{B}{r} e^{-r/L} \sum_a 4\pi r^2 dr = 4\pi \sum_a BL^2$$

$$\therefore \varphi = \frac{3Q}{4\pi\lambda_{tr}} \frac{1}{r} e^{-r/L}$$

Similarly if the neutron source is in the form of an infinite plane that emits Q thermal neutrons per unit area per second. We shall consider a slab of material infinite in the y and z directions but of finite thickness in the x-direction. Thus the steady state equation becomes.

$$\frac{d^2\varphi}{dx^2} - \frac{1}{L^2}\varphi = 0$$

The general solution of this equation can be written as

$$\phi = Ae^{x/L} + Be^{-x/L}$$

Again A = 0 if ϕ is to remain finite for all values of x. B can be evaluated, we can make use of the condition that the neutron current density J_x parallel to the source plane is J_x

Since the neutron source emits Q neutrons per sec per unit area in both + ve and -ve directions, hence by virtue of symmetry of the conditions on both sides of the source one can write.

$J_x = ½ Q = 1/3\ \lambda_{tr}\ B/L$ or $B = 3QL/2\lambda_{tr}$

If x = L, $\varphi = \frac{3QL}{2\lambda_{tr}} \frac{1}{e}$

Thus for an infinite plane as neutron source, L is the distance from the source at which the neutron flux is reduced by a factor l/e and is equal to the relaxation length which is more generally known as the thermal diffusion length. This is a measure of the air line distance travelled by neutron between the point of its origin as a thermal neutron A and the point of its absorption B.

For a point source, a relationship between the mean square distance from the source to the point of neutron absorption r^2 and L^2 can be established by the following procedure. Let the flux of neutrons at a distance r from the point source emitting neutrons at a given rate be

ϕ neutrons per unit area per second. If Σ_a is the macroscopic absorption cross section, the rate of absorption of neutrons will be $\Sigma_a \phi$ neutrons per unit volume per second. Thus the number of neutrons absorbed per second in a spherical shell of thickness dr at a distance r from the neutron source is $4\pi r^2 dr \sum\varphi$. This is the probability absorption of neutron when it is within the spherical shell of thickness dr. Therefore the mean square distance travelled by a neutron from the source to the point where it is absorbed is defined by

$$\overline{r}^2 = \frac{\int r^2 4\pi\pi^2 \, dr \sum\varphi}{\int 4\pi\pi^2 \, dr \sum\varphi}$$

$$= 6L^4/L^2 = 6L^2$$

or $\quad L^2 = 1/6\, r^2$

In other words, the square of the diffusion length is equal to one sixth the average of the square of the shortest distance that a neutron travels from the point where the neutron is emitted to where it is absorbed. The parameter L^2 is called the diffusion area.

Fast Neutron Diffusion and Fermi Age Equation

Let us now consider the diffusion of neutrons during the pre thermal or slowing down stage. By virtue of their slowing down, neutrons undergo considerable energy changes while diffusing. The neutrons change their average energy and diffuse until they reach thermal energy. In the slowing down region, neutron density per energy interval n(E) depends on the difference between the slowing down density q(E + dE) into the energy interval dE and the slowing down density q(E) out of it. This difference is zero for thermalized neutrons. In order to simplify the calculations, a continuous loss of energy for a slowing down neutron is assumed instead of actual discontinuous energy loss.

Let us investigate the neutron balance for a moderator in which there is no neutron absorption, $\Sigma_a = 0$, and in which neutrons are not being produced but diffuse through it before getting thermalized. If n is the initial number of neutrons per unit volume with energies between E and E +ΔE, the only physical processes that can cause neutron n to change are (i) diffusion of neutron into or out of the unit volume and (ii) slowing down of neutron into the energy interval ΔE and out of it. Neutron leakage by diffusion along the space co-ordinate is compensated by a neutron excess flowing along the energy co-ordinate into the energy interval dE.

In terms of neutron density the rate of diffusion is given by

$$-D\nabla^2 n = -\frac{1}{3}\lambda_{tr} v \nabla^2 n \quad \text{----------------------(1)}$$

The number of neutrons slowing down into the energy interval dE and remaining will be equal to the difference of the inflow and the outflow, that is

$$q(E+dE) - q(E) = (\partial q/\partial E)\, dE \quad \text{-----------------(2)}$$

Hence $\quad -\frac{1}{3}\lambda_{tr} v \nabla^2 n = (\partial q/\partial E)\, dE \quad$ ----------------(3)

we have

$$E\xi v \Sigma_s \nabla^2 n \quad \text{or} \quad \nabla^2 n = \nabla^2 q / E\xi v \Sigma_s \quad \text{--------(4)}$$

Substituting this value of $\nabla^2 n$ in equation (3), we get

$$\nabla^2 q = \frac{\partial q}{-\left(\lambda s \lambda_{tr} \;/\; 2\xi E\right) dE} \quad \text{---------------------------------(5)}$$

Let us introduce a new variable τ, such that

$$d\tau = -\frac{\lambda s \lambda_{tr}}{3\xi}\frac{\partial E}{E} \quad \text{--------------------------------(6)}$$

or $\quad \tau = \int d\tau = \int \frac{\lambda s \lambda_{tr}}{3\xi}\frac{\partial E}{E} = \frac{\lambda_{tr}\lambda s}{3\xi}\log\frac{E_0}{E} \quad$ -------------------(7)

Substituting this new variable in equation (5), we have

$\nabla^2 q - \partial q/\partial \tau = 0 \quad$ ----------------------(8)

This is the general equation of diffusion including all energies and is known as the Fermi age equation and the variable τ as the Fermi age or as the neutron age. Despite its name it must be emphasized that dimensions of τ are not of time as can readily be seen from equation (8), but are those of $(\text{length})^2$. It plays the same role in the slowing down equation as does the time in the heat conduction equation and is more strictly analogous to the variable kt/cp in heat conduction.

Nuclear Reactor

The atom bomb is due to an uncontrolled chain reaction. A very large amount of energy is liberated within an extremely small interval of time. Hence it is not possible to direct this energy for any useful purpose. But, in a nuclear reactor, the chain reaction is brought about under controlled conditions. If the chain reaction is put under control, after some time a steady state is established. Under a steady state, the rate of energy production also attains a constant level. Such a device in which energy is released at a given rate is known as a nuclear reactor.

The nuclear reactor consists of five main elements.

1. The Fissionable material called fuel ($^{235}U_{92}$)

2. Moderator

3. Neutron reflectors

4. Cooling system

5. Controlled and safety systems

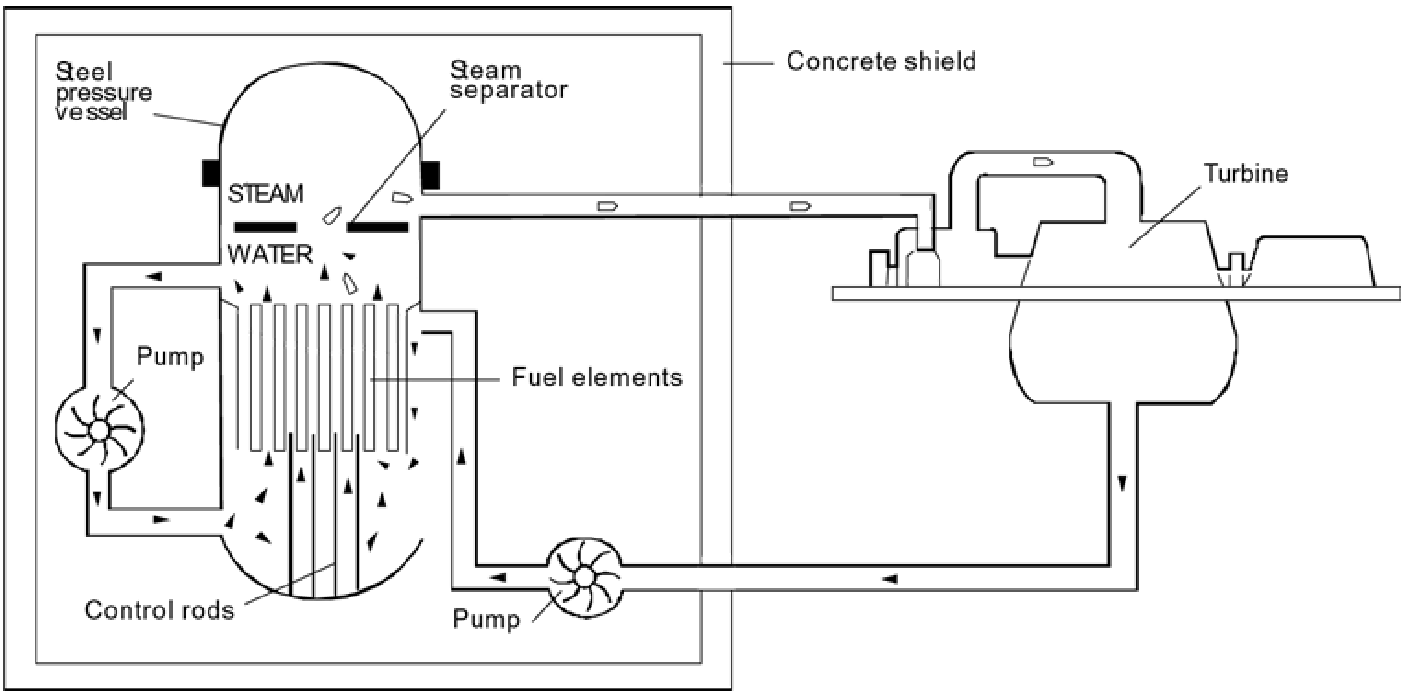


Figure: Schematic diagram of Nuclear Reactor

(1). **The fissionable substance**: During the fission of U^{235} a large amount of energy is released. The commonly used fissionable materials ae the uranium isotopes U^{233}, U^{238}, the thorium isotopes Th^{232}, and the plutonium isotopes Pu^{239}, Pu^{240} and Pu^{241}.

(2). Moderator: The function of the moderator is to slow down the highly energetic neutrons produce in the process of fission of U^{235} to thermal energies. Heavy water (D_2O), graphite, beryllium, etc., are used as a moderators. Materials used to reduce the neutron energy are known as moderators , low atomic weight and low absorption cross section for neutrons.

(3) Neutron reflector: By the use of reflector on the surface of reactors, leakage of neutrons can be very much reduced and the neutron flux in the interior can be increased. Materials of high scattering cross section and low absorption cross section are good reflectors. ideal reflectors have large energy loss by neutron per collision.

(4) Cooling system: The cooling system removes the heat evolved in the reactor core. This heat is evolved from the K.E. from the fission fragments when they are slowed down in the fissionable substance and moderator. The coolant or heat transfer agent (water, steam, He, CO_2, air and certain molten metals and alloys) is pumped though the reactor core. Then, through a heat exchanger, the coolant transfers heat to the secondary thermal system of the reactor. At ordinary temperature both light and heavy water are good coolants. For Reactor operating at high temperature , high pressure are required to prevent boiling. Liquid metals (Na) have been proposed for the use of high temperature.

(5) Control and Safety system: The control systems enable the chain reaction to be controlled and prevent it from spontaneously running away. This is accomplished by pushing control rods into the reactor core. These rods are of a material (boron or cadmium) having a large neutron absorption cross section. These rods absorb the neutrons and hence cut down the reactivity. In thermal reactors , the control rods are moved in to decrease the fission rate or the neutron flux and moved out to increase the fission rate. The safety systems protect the space surrounding the reactor against intensive neutron flux and gamma rays existing in the reactor core. This is achieved by surrounding the reactor with massive walls of concrete and lead which would absorb neutrons and gamma rays.

Nuclear Chain reaction

The schematic equation for the fission process is

$$_{92}U^{235} + _{0}n^{1} \rightarrow \; _{92}U^{236*} \rightarrow \; X + Y + \text{neutrons.} \qquad \text{-----------} \qquad (1)$$

$_{92}U^{236*}$ is a highly unstable isotope of uranium, and X and Y are the fission fragments. The fragments are not uniquely determined, because there are various combinations of fragments possible and a number of neutrons are given off. Typical fission reactions are

$$_{92}U^{235} + _{0}n^{1} \rightarrow \; _{92}U^{236*} \rightarrow \; _{56}Ba^{141} + _{36}Kr^{92} + 3_{0}n^{1} + Q \qquad \text{-------------} \qquad (2)$$

$$_{92}U^{235} + _{0}n^{1} \rightarrow \; _{92}U^{236*} \rightarrow \; _{54}Xe^{140} + _{38}Sr^{94} + 2_{0}n^{1} + Q \qquad \text{-------------} \qquad (3)$$

Where Q is the energy released in the reaction. According to equation (2) $_{92}U^{235}$ is bombarded by slow moving neuron, the nucleus becomes unstable ($_{92}U^{236*}$) and splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ releasing 3 neutrons and energy Q. According to equation (3) the number of 2 neutrons are released. So in each fission average of 2.5 neutrons are released.

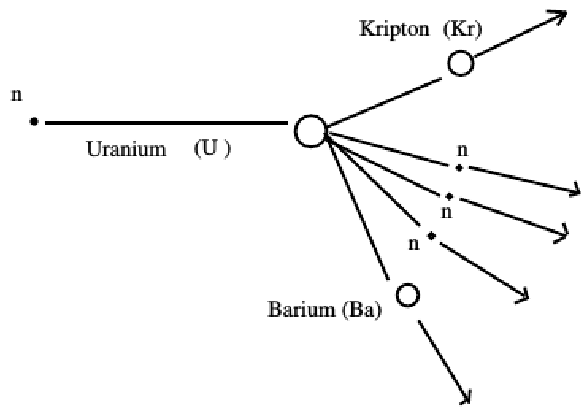


Figure : Schematic representation of Nuclear fission

In this fission process 200 MeV energy is released . These three neutrons are capable of splitting another three uranium nuclei and thus releases 9 neutrons. Like this the process spread out , and the number of neutrons and the amount of energy increases at a rapid rate. Such a self propagation process is called Chain reaction.

There are two types of chain reaction .They are controlled and uncontrolled reactions. In the controlled chain reaction the neutrons are built up certain level and there after the number of fission producing neutrons are kept constant. This principle is used in Nuclear reactors.

In the uncontrolled nuclear reaction the fission producing neutrons is allowed to multiply indefinitely and the entire energy releases all at once. This type of reaction takes place in Atom bomb

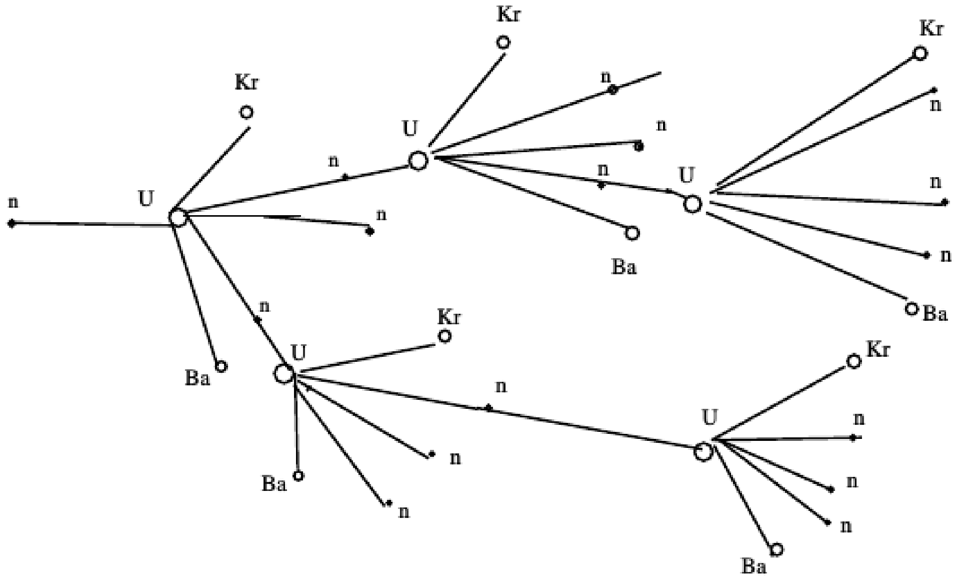


Figure : Nuclear chain reaction

Suppose a single neutron causing fission in a uranium nucleus produces 3 prompt neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing nine neutrons. These 9 neutron in turn to cause fission in nine uranium nuclei producing 27 neutrons. And so on. The number of neutrons produced in n such generations is 3^n neutrons . The ratio of secondary neutrons produced to the original neutrons is called the multiplication factor (k).

Consider 1 kg of $^{235}U_{92}$ which contains $6.023 \times 10^{26}/ 235$ or about 25×10^{23} atoms. Suppose a stray of neutron causes fission in a uranium nucleus. Each fission will release on the average 2.5 neutrons. The velocity of the neutron among the uranium atoms is such that a fission capture of thermal neutron by the $_{92}U^{235}$ nuclei takes place in about 10^{-8}s each of this fission, in turn, release 2.5 neutrons. Let us assume that all these neutrons are available for inducing further fission reactions. Let n be the number of stages of fission captures required to disrupt the entire mass of 1 kg of $^{235}U_{92}$. Then

$(2.5)^n = 25\times10^{23}$ or n = 60.

The time required for 60 fissions to take place = 60×10^{-8} s = 0.6 μs

Since each fission releases about 200 MeV of energy, this means that a total of 200 × 25 × 10^{23} = 5 × 10^{26} MeV of energy is released in 0.6 μs.

The release of this tremendous amount of energy in such a short time interval leads to a violent explosion. This results in powerful air blasts and high temperature of the order of 10^7 K or more, besides intense radioactivity.

Critical Mass for maintenance of chain reaction: Consider a system consisting of uranium (as fissile material) and a moderator. Even though each neutron that produces fission ejects 2.5 neutrons on an average, all of them are not available for further fission. The maintenance of chain reaction depends upon a favorable balance of neutrons among the three processes given below:

i) The fission of uranium nuclei which produces more neutrons than the number of neutrons used for inducing fission.
ii) Non-fission processes, including the radioactive captures of neutron by the uranium and the parasitic capture by the different substances in the substance and by impurities
iii) Escape or leakage of neutrons through the surface of the system.

If the loss of neutrons due to the last two causes is less than the surplus of neutrons produced in the first, a chain reaction takes place. Otherwise it cannot take place.

The escape of neutrons takes place from the surface of the reacting body and fission occurs throughout its volume.

∴ Escape rate varies as r^2 and production rate varies as r^3.

$$\frac{escape\,rate}{production\,rate} = \frac{1}{r}$$

The larger the size of the body, the smaller is the escape rate. Thus it is clear that by increasing the volume of the system, the loss of neutrons by escape from the system is reduced. The greater the size of the system, the lesser is the possibility of the escape of neutrons. In this case, the production of neutrons will be more than the loss due to other causes and a chain reaction can be maintained. Thus there is a critical size for the system. Critical size of a system containing fissile material is define as the minimum size for which the number of neutrons produced in the fission process just balance those lost leakage and non-fission capture. The mass of the fissionable material at this size is called the critical mass. If the size is less than the critical size, a chain reaction is not possible.

Four factor Formula

Let us study the behavior of neutrons in a reactor assembly consisting of enriched uranium mixture of the uranium isotopes $^{235}U_{92}$ and $^{238}U_{92}$ and a moderator. For simplicity we assume that the reactor is infinitely large so there is no leakage of neutrons through its surface. Let us start with the fission of a $^{235}U_{92}$ nucleus by a thermal neutron. In this fission, the number of fast neutrons produced is v. Of these, some will cause a fission in $^{238}U_{92}$ and therefore create additional fast neutron. We take account of this small increase in the neutron by introducing a factor namely **ε,** called the fast fission factor. Th number of fast neutrons available is now v**ε,**

These vε neutrons diffuse through the pile and are slowed down by collisions with moderator nuclei. However, a few of them are captured by $^{238}U_{92}$ before they are slowed down to thermal energies. This is known as resonance absorption. Therefore, we define resonance escape probability (p), as the fraction of neutrons escaping resonance absorption. Hence, the number of neutrons surviving thus far and reaching thermal energies is vεp. The value of p is usually 0.95.

Of these vεp thermal neutrons, a fraction f called the thermal utilization factor is absorbed in $^{235}U_{92}$ nuclei .While the rest are absorbed in moderator and other structural materials.

(f<1). Thus the number of $^{235}U_{92}$ nuclei undergoing fission is vεpf

When these neutrons are absorbed by $^{235}U_{92}$ in the fuel, some cause fission, where as some produce other kinds of reactions such as capture. The fraction of neutrons causing fission is given the ratio of fission cross-section σ_f to the total absorption cross-section σ_a, the number of second generation thermal neutrons causing fission of

$$^{235}_{92}U = v\varepsilon pf\,\frac{\sigma_f}{\sigma_a}$$

Now, $v\frac{\sigma f}{\sigma a} = \eta$ is the average number of fission neutrons released per neutron absorbed by a fissionable nuclide.

The number of fissions produced to a single neutron is $K = \eta\varepsilon pf$

This is the *multiplication factor.* We have assumed that the reactor assembly is of an infinite size so that there is no leakage of neutrons. $K_\infty = \eta\varepsilon pf$

This relation is called the *four-factor formula*. In designing a nuclear reactor, the fundamental problem is to maintain *K* at 1 in order to achieve a self-sustaining chain reaction. Of the four factors, η and ε are fixed for a given fuel. The other two factors *p* and *f* are made as large as possible by properly designing the reactor geometry, arranging the fuel, and choosing the moderator.

Classification of Nuclear Reactors

Pressurized Water Reactor (PWR) If the pressure on the water surface is increased, its boiling point increases so that it can have more quantity of heat energy per unit mass. This principle is utilized in PWR.

The fuel is in the form of uranium oxide (UO_2). It is sealed in long, thin zirconium-alloy tubes that are assembled together with movable control rods in to a core. The core is enclosed in a steel pressure vessel. Light water is used as coolant and moderator.

The water that circulates past the core is kept at a sufficiently high pressure, about 150 atmospheric pressure , to prevent boiling. The water enters the pressure vessel at about

280°C and leaves at about 320°C. The water then passes through a heat exchanger (steam generator) which produces steam. The resulting steam then passes out of the containment shell. The containment shell serves as a barrier to protect the outside world from accidents to the reactor. The high pressure steam drives a turbine. The turbine drives an electric generator and produces electrical power. Steam after running the turbine gets condensed into water in the condenser. This water is again circulated through heat exchanger by means of a pump.

Boiling Water Reactor (BWR)

Uranium is used as fuel in BWR. Water is used as a coolant and moderator. Circulating water absorbs heat produced by fission of the fuel and gets converted into high pressure steam in the reactor core itself. This steam runs the turbine which in turn produces electric power. Steam after running the turbine gets condensed into water in the condenser. This water is again circulated into the reactor core by using a pump. The steam leaving the reactor may be slightly radioactive. Therefore, the pipe lines and turbine assembly are properly shielded.

Fast Breeder Reactor

If a thermal rector core with U^{235} fuel is surrounded by a blanket of a fertile material like U^{238}, U^{238} can be converted into fissile fuel. Reactors of this type are called fuel producing reactors. The reactions are as follows:

$$_{92}U^{238} + {}_{0}n^{1} \rightarrow {}_{92}U^{239} + \gamma$$

This is followed by $_{92}U^{238} \rightarrow {}_{93}Np^{239} + \beta^{-}$

$_{93}Np^{239}$ is also radioactive. It emits a β particle to form plutonium.

$$_{93}Np^{239} \rightarrow {}_{94}Pu^{239} + \beta^{-} + \gamma$$

This process of producing one type of fissionable material ($_{94}Pu^{239}$) from a non-fissionable material (U^{238}) is called the breeding and reactor a breeder reactor.

The breeding reactors for the fertile material $_{90}Th^{232}$ are

$_{90}Th^{232} + n \rightarrow {}_{90}Th^{233*} + \beta^- \rightarrow {}_{91}Pa^{233} + \beta^-$

A liquid-sodium-cooled fast breeder reactor. The core may consist of ^{235}U and ^{239} Pu, while the blanket contains the fertile ^{238}U that will breed into fissionable material. Liquid sodium is very effective as a medium for heat transfer. But its chemical reactivity and the radioactivity induced in it by neutron irradiation pose serious problems. To prevent contaminating the generating system in the event of a sodium leak, an intermediate liquid sodium cooling loop is between the primary loop that passes through the reactor core and the steam generator.

Uses of nuclear reactor:

1. Nuclear power: nuclear reactors are used in the production of electric energy.
2. Production of radio isotopes: nuclear reactors are useful in producing a large number of radio isotopes. To produce radio isotopes, a suitable compound is drawn into the center of the reactor core where the flux of neutrons may well be more than $10^{16}/m^2/sec$. sodium 24 is manufactured in this way.

$$_{11}Na^{23} + {}_0n^1 \rightarrow {}_{11}Na^{24*}$$

3. Scientific research: Reactors produce a number of radioactive materials needed for research purposes. The reactors provide a huge source of neurons. Using these neutrons, several useful radioisotopes have been artificially produced and several nuclear reactions have been studied. Effects of neutron in biological tissues is also studied. Radiation damage is also studied.

UNIT V : NUCLEAR PARTICLES

Classification of elementary particles – particle interaction – conservation laws- leptons Hardons-pion - muons – mesons – hyperons - strange particle – CPT theorem -- quark model- Elementary particle symmetries SU(2) and SU(3) symmetry

Classification of Elementary Particles

After studying the structure of atoms, one gets the impression that perhaps electron, proton and neutron are the only building blocks of matter. Studies made partly on high energy cosmic ray particles revealed the existence of numerous new nuclear particles. They are the subatomic or elementary particles . These particles are elementary in the sense that they are structure less. The following figure shows the classification of elementary particles. The elementary particles are separated into two general groups called bosons and fermions. Bosons are particles with intrinsic angular momentum equal to an integral multiple of ħ. Fermions are all those particles in which the spin is half integral.

Baryons: Proton and particles heavier than protons form this group. Protons and neutrons are called nucleons and the rest are called hyperons. Every baryon has an antiparticles. I a number called the baryon number is +1 is assigned to baryons and number -1 is assigned to antibaryons, then in any closed system interaction or decay the baryon number does not change. This is the law of conservation of baryons. Hyperons is a special class of baryons characterized by a time decay of the order of 10^{-10} seconds and mass value intermediate between those of then neutron and deuteron. Their decay time is very much greater than the time of their formation(10^{-3}).

Leptons : This group contains electron , photon, neutrino and muon

Mesons : The rest mass of these particles varies between about 250 m_e and 1000 m_e. The mesons are the agents of the interaction between particles inside the nucleus. Pions, kaon and η- mesons are together called **mesons.** Baryons and mesons are jointly called **hadrons** and are the particles of strong interaction

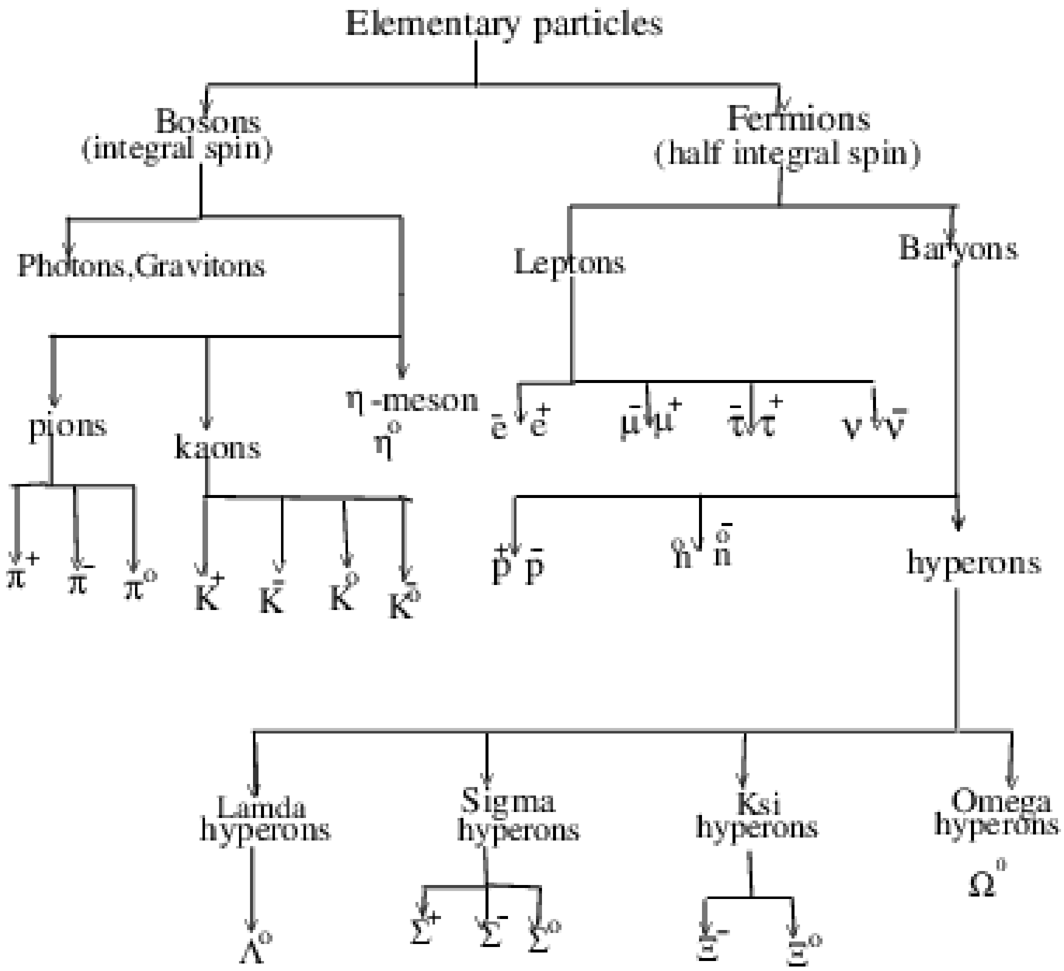


Figure: Classification of Elementary Particles

Particles and Anti-particles

Electron and Positron. The positron and the electron are said to be antiparticles. They have the same mass and the same spin but opposite charge. They annihilate each other with the emission of photons, when they come in contact with each other. Existence of an antiparticle for the electron was actually predicted by Dirac. Positron was discovered by Anderson in 1932.

Proton and antiproton. The antiproton, was established in 1955 by Segre, Chamberlain, and their collaborators. Antiprotons were produced by bombarding protons in a target with 6-GeV protons, thereby inducing the reaction.

$$p + p\ (\text{+energy}) \rightarrow p + p + p + \overline{p}$$

Antiprotons interact strongly with matter and annihilate with protons.

$$P + \bar{p} \rightarrow \pi^{+} + \pi^{-} + \pi^{+} + \pi^{-} + \pi^{0}$$

Neutron and antineutron. The antiparticle of neutron, *antineutron*,discovered in 1956 by Cork, Lamberton and Wenzel.The nature of the antineutron is not very well known . Both neutron and antineutron have zero charge and the same mass. However, since neutron is supposed to have a certain internal charge distribution , it is expected that the antineutron has an internal charge distribution opposite to that of the neutron. Antineutron is quickly annihilated, either by a proton or a neutron, usually with the production of several pions. If an antineutron is not annihilated by a nucleon , it decays by the reaction.

$$\bar{p} + p \rightarrow \bar{n} + n$$

Neutrino and antineutrino. The neutrino has a finite energy and momentum in flight. It travels with the speed of light *c*. It does not cause ionization on passing through matter.

The antiparticle of neutrino is antineutrino. The distinction between the neutrino v and antineutrino $\bar{v}$ is a particularly interesting one. The spin of the neutrino is opposite in direction to the direction of its motion. The neutrino spins-counter clockwise. But the spin of the antineutrino is in the same direction as its direction of motion, it spins clockwise. Thus the neutrino moves through space in the manner of a left-handed screw, while the antineutrino does so in the manner of a right-handed screw. Thus neutrino possesses a " left-handed" helicity; The antineutrino possesses a " right-handed " helicity, i.e., A neutrino and antineutrino differ only in the sense of their helicity.

It is customary to The particle accompanying a positron a neutrino, v, while that accompanying an electron is called an antineutrino. Because of its lack of charge and magnetic moment, a neutrino has essentially no interaction with matter. This interaction is extremely weak.

$$\mathbf{n} \rightarrow \mathbf{p} + \mathbf{e}^{-} + \bar{v}$$

$$\mathbf{p} \rightarrow \mathbf{n} + \mathbf{e}^{+} + \nu$$

Antimatter

In Atomic Physics, it has long been useful to consider an atom as composed of extra nuclear electrons and a nucleus. A positron and an anti proton could form an atom of anti hydrogen. Anti hydrogen would have a spectrum similar to that of ordinary hydrogen. Indeed, from a collection of anti protons, anti neutrons, and positrons, everything were made of antiparticles. Particle-antiparticle annihilation would occur with a tremendous release of energy.

The Fundamental Interaction

The fundamental interactions may be defined as the fundamental forces that act between the elementary particles of which all matter is assumed to be composed. The fundamental interactions may be classified as (i) Strong interaction (ii)Electromagnetic interaction (iii) Weak interaction and (iv) Gravitational interaction. All known processes in nature (from sub nuclear to extra galactic ie. microscopic to macroscopic) can be considered as a manifestation of one or more of these interactions. The following table shows the particle exchanged in each of these interactions .

Table: Particles exchanged in the interaction

Interaction	*Particles Exchanged*
Strong	Mesons
Electro-magnetic	Photons
Weak	Intermediate bosons
Gravitational	Gravitons

The gravitational forces are not significant for elementary particles and nuclear physics. Out of the rest three ,weak forces has a very short range ($<10^{-17}$ m) and is extremely feeble compared to strong and electromagnetic forces.

1. **Strong interaction**. The strong interaction is the forces which hold nucleons together (nuclear forces) in the atomic nucleus. The strong nuclear interaction is Independent of the electric charge. The range of these interactions is about 10^{-15}m. Time interval of such on interaction is roughly10^{-23} s.
2. **Electromagnetic interaction**. It operate on all charged particles. Thus electro-magnetic interactions are charge dependent. The range is infinite and the interaction works through the photon. The formation of electron-position pair from gamma ray is an example of electromagnetic interaction.
3. **Weak interaction**. All interactions take place in times of about 10^{-23}s their decay takes place in time of about 10^{-10}s. Since particles take long time to respond to such an interaction, force involved must be very weak compared with strong nuclear forces. The range of such an interaction is less than 10^{-17}m. The characteristic time of this interaction is 10^{-8} s. The weak interaction is responsible for the decay of strange and non-strange particles and for non-leptonic decays of strange particles.

   Beta decay is an example of weak interaction $n \rightarrow p + e^{-} + \bar{\nu}$

4. **Gravitational interaction**. This interaction manifests itself as a long range force of attraction between all elementary particles. The first force that any of us discover is of gravity. It holds the moon and earth together, keeping the planets in their solar orbits and binds stars to from our galaxy. Newton gave a formula $F= G\, m_1 m_2/r^2 \sim 2\text{x}\ 10^{-34}$ newton. And gravitational attraction is only about $2\text{x}\ 10^{-49}$ joule. Hence we see that it plays no role in particle reactions. It is the weakest of the four types of interactions. It has infinite range. Gravitation can be explained in terms of the interactions of '*gravitons*'. Their mass must be zero, and therefore, their velocity must be that of light. The gravitational force does not depend on the colour, size,charge,velocity, spin and angular orientation but depends on the magnitude of inertia. As the gravitational field is extremely weak the gravitons cannot be detected in the laboratory.

Conservation laws of Elementary Particles

In classifying the various elementary particles, several discrete quantum numbers are used. We are already familiar with two such quantum numbers, namely those that describe a particle's charge and spin. These quantum numbers, specify measurable physical properties and are always conserved. We know that all elementary charges are 0 or 1. Particles with integer spin obey the Bose-Einstein statistics and are called bosons. Particles with half odd integer spins obey the Fermi-Dirac statistics and are called fermions.

1. **Baryon number**. Each **baryon** is given a baryon number B=1,each corresponding anti baryon is given a baryon number B= - 1. All other particles have B = 0. The law of conservation of baryons states that the sum of the baryon number of all the particles after a reaction or decay must be the same as before. This rule ensures that a proton cannot change into an electron , even though a neutron can change into a proton. Baryon conservation ensures the stability of the proton against decaying into a particle of smaller mass. All normal baryons such as p^+, n^0, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, Ω have the baryon of +1. All the corresponding anti particles known as anti baryons have the baryon number -1.

All the mesons have baryon number 0.

$$\mathbf{\Lambda^0} \rightarrow \mathbf{p^+} + \pi^- \quad \text{ie. } [\, 1= 1+ 0 \,]$$

Another example for the conservation of baryon number is

$$n \rightarrow \mathbf{p^+} + \mathbf{e^-} + \bar{\nu} \qquad [\, 1 = 1 +0+ 0]$$

Hence the baryon number is conserved

2. **Lepton number**. The **Leptons** are supposed to possess a property called *Lepton number* (L). Since the neutrinos associated with electrons and with muons are recognized as different, we introduce two lepton numbers L_e and L_μ both of which must be conserved separately in particle reaction and decays. The number L_e = 1 is assigned to the electron and the e-neutrino and L_e = - 1 to their antiparticles. All other particles have L_e = 0. Also the number L_μ = 1 is assigned to the **muon** and the **μ – neutrino** and $L\mu$ = - 1 to their antiparticles.

$$\pi^- + p^+ \rightarrow \Lambda^0 + K^0 \qquad [\, 0+0 =0 +0]$$

3. **Strangeness number**. They were produced by high energy reactions but always in pairs i.e., if one particle of some kind is produced then simultaneously another different particle is also emitted. Associated production of **kaons** and **hyperons** which are known as strange particles. Decay through strong interactions in a very short time but this is not observed. Instead they decay slowly. Because of this strange behavior they were called as strange particles.

$$K^0 \rightarrow \pi^+ + \pi^- \qquad [+1 = 0+0]$$

Here the LHS is 1 and RHS is 0. So the reaction is not conserved for strangeness

It is found that S is conserved in all processes mediated by the strong and electromagnetic interactions. The multiple creation of particle with S not equal to 0 is the result of the conservation principle. An example is the proton-proton collision:

$$P^+ + P^+ \rightarrow \Lambda^0 + K^0 + P^+ + \pi^+$$

strangeness number S ; 0 + 0 → (-1) + 1 + 0 + 0

On the other hand, S can change in an event governed by the weak interaction. The decays of kaons and hyperons proceed via the weak interaction and accordingly are extremely slow. Even the weak interaction, however, is unable to permit S to change by more than ± 1 in a decay.

$\Sigma^+ \rightarrow \Lambda^0 + e^+ + v_e$ [-1 = -1 + 0+0] ; ΔS=0

$\Sigma^- \rightarrow n + e^- + \bar{v}_e$ [-1= 0+0+0] ; ΔS= -1

$\pi^- + p \rightarrow \Lambda^0 + K^0$ [0+ 0 = -1 +1] ; ΔS=0

4. Isospin and Isospin quantum number.

As far as strong interactions are concerned, the neutron and the proton are two states of equal mass of a nucleon doublet. It is found that particles occur in multiplets. For example, singlets η^0 (**eta-meson**), Ω^-**(omega hyperons)** , Λ^0(**lamda hyperon**). Doublet : *p*, *n*, triplet, π^+, π^-, π^0 **(pions or Pi-mesons)** . It is natural to think of the members of a multiplet as representing different charge states of a single fundamental entity. It has proved useful to categorize each multiplet according to the number of charge states it exhibits by a number I such that the multiplicity of the state is given by 2 *I*+ 1. Thus the nucleon multiplet is assigned *I* = ½, and its 2 x ½ + 1 = 2 states are the neutron and the proton. The pion multiplet has I = 1, and its 2 x 1 + 1 = 3 states are π^+, π^-, π^0

Isospin can be represented by a vector *I* in "isospin space" whose component in any specified direction is governed by a quantum number customarily denoted I_3. The possible values of I_3

Class	*Name*	*Symbol*	*Spin*	*B*	*Le*	*Lμ*	*S*	*Y*	*I*
LEPTON	e- neutrino	v_e	½	0	+ 1	0			
	μ – neutrino	v_μ	½	0	0	+1			
	Electron	e^-	½	0	+ 1	0			
	Muon	μ^-	½	0	0	+1			
MESON	Pion	$\bar{\pi}, \pi^+$ π^0	0	0	0	0	0	0	1
	Kaon	K^+ K^0	0	0	0	0	+ 1	+1	1/2
	η meson	η^0	0	0	0	0	0	0	0
BARYON	Nucleon	*p,n*	½	+ 1	0	0	0	+1	1/2
	Λ hyperons	Λ^0	½	+ 1	0	0	-1	0	0
	Σ hyperons	Σ^+ Σ^0 Σ^-	½	+ 1	0	0	-1	0	1
	Ξ hyperons	Ξ^0 Ξ^-	½	+ 1	0	0	-2	-1	1/2
	Ω hyperons	Ω^-	3/2	+ 1	0	0	-3	-2	0

are restricted to $I, I-1, I-2$…….., 0, ….. – ($I-1$), - I. Hence I_3 is half-integral, if I is half-integral and integral or zero if I is integral

For the nucleon, I = ½ which means that I_3 can be either +1/2 or -1/2 ; The former is taken to represent the proton and the later neutron.

Similarly, for the pion triplet I = 1 giving I_3 = + 1 for π^+, I_3 = 0 for π^0 and – 1 for π^-. The charge of a meson or baryon is related to its baryon number **B**, its strangeness number **S**, and the component $\boldsymbol{I_3}$ of its isospin by the formula

$$q = e\left(1_3 + \frac{B+S}{2}\right) = e\left(I_3 + \frac{Y}{2}\right)$$

5. Hypercharge.

A quantity called **hypercharge (*Y*)** is conserved in strong interaction. Hypercharge is equal to the sum of the strangeness and baryon numbers of the particle , ***Y = S + B.*** For mesons B=0, so the hypercharge equals the strangeness.

Conservation Laws and Symmetry

A very important set of conservation laws is related to symmetries involving parity

(*P*), charge conjugation (*C*), and time reversal (*T).*

Charge conjugation symmetry. Charge conjugation is the act of symmetry operation in which every particle in a system is replaced by its antiparticle. If the antisystem, or antimatter counterpart exhibits the same physical phenomena, then charge parity (C) is conserved. For example, if in a hydrogen atom, the proton is replaced by an antiproton and the electron is replaced by a positron, then this antimatter atom will behave exactly like an ordinary atom. In fact C is not conserved in the weak interaction.

Conservation of parity : Parity relates to the symmetry of the wave function that represents the system. If the wave function is unchanged, when the coordinates (x, y, z) are replaced by (- x, - y, - z), then the system has a parity of + 1. If the wave function has its sign changed, when the coordinates are reversed, then the system has a parity of -1. If we write ψ (x, y, z) = P ψ (-x, -y, -z), we can regard P as a quantum number characterizing ψ whose possible values are +1 and -1. During a reaction in which parity is conserved, the total parity number does not change.

Changing the coordinates (x, y, z) into (- x, - y, - z) convents a right-handed coordinate system into a left-handed coordinate system. In terms of symmetry, the meaning of conservation of parity is that in any situation where parity is conserved, the description of the reaction will not be changed if the word “left” is changed to the word “right” and vice versa. This means that such reactions can provide no clue that will distinguish between the directions right and left. Prior to 1956 it was believed that all reactions in nature obeyed the law of conservation of parity. However, Yang and Lee pointed out that in reactions involving the weak interaction, parity was not conserved, and that experiments could be devised that would absolutely distinguish between right and left. Indeed parity conservation is found to hold true only in the strong and electromagnetic interactions.

Time reversal symmetry. Time parity *T* describes the behavior of a wave function when *t* is replaced by –*t*. The symmetry operation that corresponds to the conservation of time parity is time reversal. Time reversal symmetry implies that the direction of time is not significant, so that the reverse of any process that can occur is also a process that can occur. In other words, if symmetry under time reversal holds, it is impossible to establish by viewing it whether a motion picture of an event is being run forward or backward. Prior to 1964, time parity *T* was considered to be conserved in every interaction. It was discovered in 1964 that one form of the K^0 , kaon can decay into $\pi + \pi^-$ which violates the conservation of T. The symmetry of phenomena under time reversal thus does not seem to be universal.

CPT Theorem: The combined symmetry operation in which the antimatter mirror-image of a system is run in reverse allows a test of *CPT* invariance. All the evidence supports the conservation of *CPT*. The conservation of *CPT* means that for every process there is an antimatter mirror-image counterpart that takes place in reverse. This particular symmetry seems to hold for all interactions, even though its component symmetries sometimes fail individually.

The Quark Model

Murray Gell-Mann and G. Zweig proposed the quark model in 1964. This theory is based on the idea that the hadrons are built up from a limited number of “fundamental” units, which have acquired the name quarks. The original three quarks were labeled u (for “*up”*), d (for”*down”*) and s (for “*strange”*).

Quark	symbol	charge	spin	B	S	I	I_3
Up	u	+2/3	½	1/3	0	½	+1/2
down	d	-1/3	½	1/3	0	½	-1/2
strange	s	-1/3	½	½	-1	0	0

u quark has electric charge + $\frac{2}{3}e$ and strangeness 0.

d quark has electric charge - $\frac{1}{3}e$ and strangeness 0.

s quark has electric charge - $\frac{1}{3}e$ and strangeness – 1.

Each quark has a baryon number of B = 1/3.

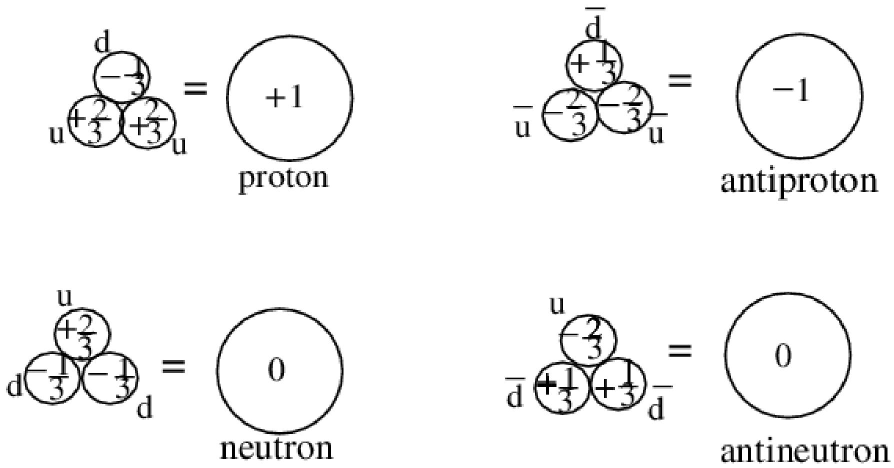


Figure: Constituents of proton and neutron in terms of quark

Each quark has an antiquark associated with it ($\overline{u},\overline{d},\overline{s}$). The magnitude of each of the quantum numbers for the antiquarks has the same magnitude as those for the quarks, but the sign is changed.

Compositions of hadrons according to the quark model

Hadrons may be baryons or mesons. A baryon is made up of three quarks. For example, the proton is made up of two *u* quarks and one *d* quarks *(uud).* For these quarks, the electric charges are + 2/3, + 2/3, and – 1/3, for a total value of + 1. The baryon number are + 1/3, + 1/3 and + 1/3, for a total of + 1. The strangeness numbers are 0, 0 and 0 for a total strangeness of 0. All are in agreement with the quantum numbers for the proton. quark models of the proton, antiproton, neutron and antineutron. Electric charges are given in units of *e.*

A meson is made up of one quark and one antiquark. For example, the π^+ meson is the combination of one quark either *u* or *d* and one antiquark either $\overline{u},\overline{d},$. The Electric charges of these quarks are + 2/3 and + 1/3 for a total of +1, for a total of baryon number of 0. The strangeness numbers are 0 and 0 for a total of 0. All of these are in agreement with the quantum numbers for the pi-meson. Quarks all have spins of ½, which accounts for the observed half-integral spins of baryons and the 0 or 1 spins of mesons.

All known hadrons can be explained in terms of the various quarks and their antiquarks. Quark contents of five hadrons and how they account for the observed charges, spins, and strangeness numbers of these particles.

Hadron	*Quark content*	*Baryon number*	*Charge, e*	*Spin*	*Strangeness*
π^+	$u\overline{d}$	$\frac{1}{3}-\frac{1}{3}=0$	$+\frac{2}{3}+\frac{1}{3}=+1$	↑↓=0	0 + 0 = 0

$K+$	$u\bar{s}$	$\frac{1}{3}-\frac{1}{3}=0$	$+\frac{2}{3}+\frac{1}{3}=+1$	↑↓=0	0 + 1 = + 1
$p+$	uud	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$+\frac{2}{3}+\frac{2}{3}+\frac{1}{3}=1$	↑↑↓= $\frac{1}{2}$	0 + 0 + 0 = 0
n^0	ddu	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}=0$	↓↓↑= $\frac{1}{2}$	0 + 0 + 0 = 0
Ω -	sss	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=+1$	$-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}=-1$	↑↑↑= $\frac{3}{2}$	-1 -1 -1 = -3

Coloured quarks and gluons: There were problems with the quark model, one of them being Ω^- hyperon. It was believed to contain three identical *s* quarks (*sss*). This violates the Pauli exclusion principle, that prohibits two or more fermions from occupying identical quantum states. The proton, neutron, and others with two identical quarks would violate this principle also. We can resolve this difficulty by assigning a new property to the quarks. We can regard this new property as an additional quantum number that can be used to label the three otherwise identical quarks in the Ω^-. If this additional quantum number can take any one of three possible values, we can restore the Pauli's principle by giving each quark has a different value of this new quantum number, which is known as colour. The three colours are labeled red (*R*), blue (*B*), and green (*G*). The Ω^- for example, would then S_R , S_B ,S_G. The antiquark colours are antired (*R*) antiblue (*B*) and antigreen (*G*).

An essential component of the quark model with colour is that all observed meson and baryon states are "colourless", i.e., either colour, anticolour combinations in the case of mesons, or equal mixtures of R, B and G in the case of baryons.

Since hadrons seem to be composed of quarks, the strong interaction between hadrons should ultimately be traceable to an interaction between quarks. The force between quarks can be modeled as an exchange force, mediated by the exchange of mass less spin –1 particles called gluons. Eight gluons have been postulated. The field that binds the quarks is a colour field. Colour is to the strong interaction between quarks as electric charge is to the electromagnetic interaction between electrons. It is the fundamental strong "charge" and is carried by the gluons. The gluons must therefore be represented as combinations of a colour and a possibly different anticolour. The gluons are massless and carry their colour and anticolour properties just as other particles may carry electric charge. For example, a gluon RB being exchanged by red and blue quarks. In effect the red quark emits its redness into a gluon and acquires blueness by also emitting antiblueness. The blue quark, on the other hand, absorbs the $R\overline{B}$ gluon, canceling its blueness and acquiring a red colour in the process.

Charm, Bottom, and Top. The charmed quark was suggested to explain the suppression of certain decay processes that are not observed. With only three quarks, the processes would proceed at measurable rates and should have been observed. The charm quark has a charge of $\frac{2}{3}e,$ strangeness 0 and a charm quantum number of + 1. Other quarks have 0 charm.

Generation	Quark	Symbol	Charge, e	Strangeness	Charm
1	Up	u	$+\frac{2}{3}$	0	0
	Down	d	$-\frac{1}{3}$	0	0
2	Charm	c	$+\frac{2}{3}$	0	+1
	Strange	s	$-\frac{1}{3}$	-1	0
3	Top	t	$+\frac{2}{3}$	0	0
	Bottom	b	$-\frac{1}{3}$	0	0

Quantum Numbers – The quarks have quantum numbers. The *s*-quark has a quantum number called strangeness. The *C*, *B* and *T* quantum numbers are conserved in the strong and electromagnetic interactions and change by one unit in the weak interactions. This means that the number of quarks minus antiquarks for each *s*, *c*, *b* and *t* must remain constant in strong and electromagnetic interactions, whereas in the weak interaction there is a change of quark flavor with the preferred sequence $t \rightarrow b \rightarrow c \rightarrow s$. Since three of the quarks are needed to make a baryon, therefore, the baryon number is 1/3 for all the quarks. The quarks quantum numbers are summarized in table .

Table: Quark quantum numbers and properties

Quantum Number	***u***	***d***	***s***	***c***	***b***	***t***
Charge	2/3	-1/3	-1/3	2/3	-1/3	2/3
Mass (GeV/c^2)	0.39	0.39	0.51	1.55	~ 5	~ 30
Spin in ***h***	1/2	1/2	1/2	1/2	1/2	1/2
Isospin ***I***	1/2	1/2	0	0	0	0
Isospin Component I_3	1/2	-1/2	0	0	0	0
Baryon number ***B***	1/3	1/3	1/3	1/3	1/3	1/3
Strangeness ***S***	0	0	-1	0	0	0
Charm $\mathcal{C}$	0	0	0	1	0	0
Botton $\mathcal{B}$	0	0	0	0	-1	0
Top $\mathcal{T}$	0	0	0	0	0	1

The isospin quantum number T is ½ and therefore T_3=1/2 and -1/2 for the up and down quarks respectively. The quantum number S of strange quark and of beauty quark is -1. It is 1 for the charm and top quarks. The hypercharge is a quantum number related to quark strangeness and baryon number, whereas the isospin is a quantum number related to the *u*-*d* quark difference. The colour quantum number breaks the degeneracy and allows up to three quarks of the same flavor to occupy a single quantum state.

Quark Masses - Among the six quarks, the least massive members are the *u* and *d* quarks, each of same mass, around 0.39 GeV/c^2. The lighest baryons, nucleons, Δ particles, and the lightest mesons, pions must therefore be exclusively made of these two quarks. The s quark is more massive, around 0.51 GeV/c^2. It carries a quantum number called strangeness and a necessary constituent of particles called strange particles (with non-zero strangeness), such as K-mesons, and baryon A. The *c*-quark is even more massive, having rest mass around 1.65 GeV/c^2. The *b*-quark has a rest mass around 5 GeV/c^2.

Mesons and Baryons – All hadrons are made of six quarks and their antiquarks. The properties of the quarks is inferred from the properties of mesons and baryons. To know the masses of the quarks from the known hadron masses, we need to know the strength of the interaction between quarks in the hadron. The hadrons are subdivided into two classes, baryons and mesons. Baryons are Fermions, this implies that quarks are also the fermions. Since the quark cannot exist as a free particle, the lightest fermion in the hadron family must therefore be made of three quarks. Thus

$| p> = | uud >$ and $| n > = | u\, dd>$.

$$Q_p = \frac{2}{3} + \frac{2}{3} + \left(\frac{-1}{3}\right) = 1; \qquad Qn = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0 .$$

Elementary Particle Symmetries

It has been found that, in general, the conservation law represents an invariance which corresponds in an appropriate symmetry operation. The set of operators that represents the symmetry constitutes the group from which the theory gets its name. The irreducible representations of a group consist of a number of states, quantities or objects to which the symmetry operations are applicable. Thus the appropriate group operation can transform any one of these states into another in the same representation. The fundamental representation is the one containing the smallest number of states for the particular group.

Linear momentum is conserved if the system is in variant with respect to displacement in space. Angular momentum is conserved if invariance is with respect in angular displacement and energy is conserved if it is with respect to time.

The simplest unitary group *U* (1) contains transformations which add a phase factor only to particle wave functions. The invariance under such transformations gives conservations of charge Q, baryon number B, lepton number L and hypercharge Y

Unitary Symmetry (SU(2) Symmetry)

We know that proton and neutron are identical as far as the nuclear force is concerned, but differ in their electromagnetic interactions. Thus, it is possible to imagine a group of symmetry operators which could transform a neutron into a proton (or proton into a neutron) in the absence of an electromagnetic field. The proton and neutron would then form the fundamental representations of the group. The existence of such symmetry implies that something remains constant under the strong interaction. This is known as isospin and is ½ for proton as well as for neutron. The component of the isospin, T_3 is + ½ for the proton and - ½ for the neutron. The operators of the symmetry group thus change the co-ordinates of isospin in such a way as to reverse the sign of T_3. It can also be expressed as: the strong interactions are assumed to be invariant under rotations in the isotopic spin space.

The particular symmetry group applicable to isospin conservation is a form of unitary symmetry known as U(2), which can be expressed by a set of 2 x 2 matrices. This group may be reduced to a special unitary group SU (2), which is also written as SU_2. It is special because a restriction reduces by unity the number of operators in the group. The two dimensions refer to the two basic states which make up the fundamental representation in this case. The restriction of reducing the number of operators 2 x 2 = 4 to three. The group is then said to have three generators.

By the use of the algebra of the *SU(2)* group it can be shown that all irreducible representations of the symmetry group consist of a multiplet of 2T + 1 states. All the members of the multiplet have the same isospin T and are essentially identical except for charge. If the symmetry was exact, i.e. isospin is strictly conserved, the components of a multiplet would differ in charge and T_3 . The *SU(2)* symmetry is violated by the electromagnetic interaction for which conservation of isospin is not applicable.

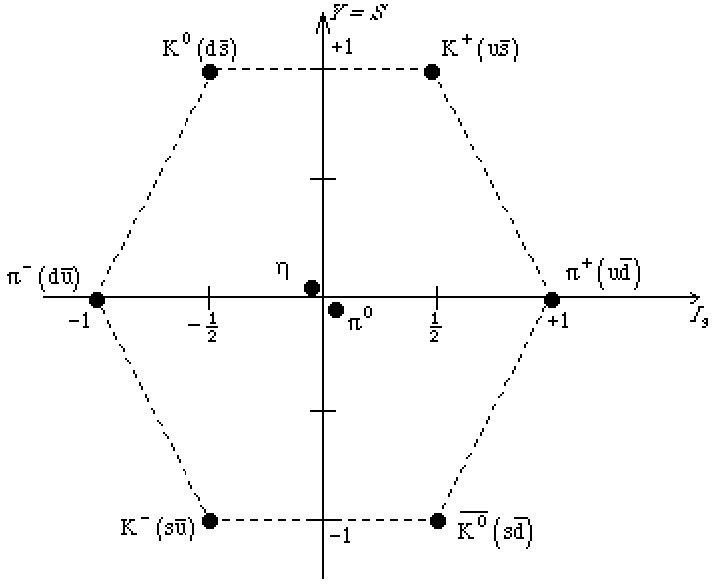
Eightfold Way (SU (3) Symmetry)

Since *SU(2)* group cannot accommodate the hyper charge quantum number, the more general theory SU (3) has been used which also includes SU(2). SU(3) stands for special unitary group in three dimensions. The term, three dimensions refers to the three basic states which make up the fundamental representation in this case. In a three dimension unitary group there are, in general 3 x 3 = 9 operators but the restriction of "special" reduces the number to eight. The group is then said to have eight generators. Three of the generators apply to three components of isospin, as in SU(2) and a fourth is associated with hypercharge. The remaining four also involve hypercharge in a different way.

Application of the group algebra showed that the SU (3) symmetry should give rise to six super multiplets, containing 1, 8, 8, 10, $\overline{10}$ and 27 members. The $\overline{10}$ multiplet is equivalent to the 10 but with hypercharges of opposite signs. In each of these super multiplets the parity and intrinsic spin of members are the same, while the hypercharge and the isotopic spin are not same. Among above mentioned groups, 8 and 10 member groups are of particular interest.

In the case for B=0 we may form particles anti particle states in a 3x 3 array . The matrix may contain a total of 8 states and is known as on octet, since mesons are formed from fermions particle-antiparticle pairs, hence have a odd parity. These 8 particles with B=0 and $J^{\pi} = 0^{-}$ should be arranged as follows and the schematic representation is given in the figure.

One triplet with	Y=0, T=1		$\pi^{-}, \pi^{0}, \pi^{+}$
One double with	Y=1, T= ½		K^0, K^+
One double with	Y= -1, T= ½	8 members	$\overline{K^0}, K^-$
One single with	Y= 0, T=0		η^0



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